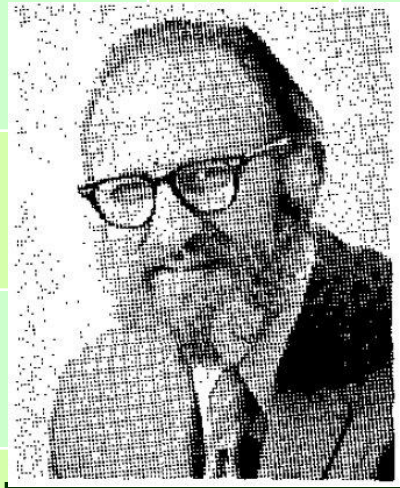
Minimización de Funciones Booleanas

Mapas de Karnaugh

No me preocupa que aún no dominen los temas, ***porque están aquí para aprender.***

Lo que realmente me inquieta es cuando **no muestran curiosidad ni ganas de hacerlo**

JAGG



Maurice Karnaugh

Nació el 4 de octubre de 1924 en New York. Fue un Físico americano que tuvo como principal aporte En 1953 la creación de los Mapas de Karnaugh o Diagrama de Veitch, cuya función es la de minimizar funciones algebráicas booleanas.

Graduado de Ingeniero de Telecomunicaciones en la universidad de Yale en 1952. Es actualmente gobernador emérito del ICCC (International Council for Computer Communication).

Trabajó como investigador en los Laboratorios Bell desde 1952 a 1966 y en el centro de investigación de IBM de 1966 a 1993.

Impartió clases de informática en el Politécnico de Nueva York de 1980 a 1999, y desde 1975 es miembro del IEEE (*Institute of Electrical and Electronics Engineers*) por sus aportaciones sobre la utilización de métodos numéricos en las telecomunicaciones.

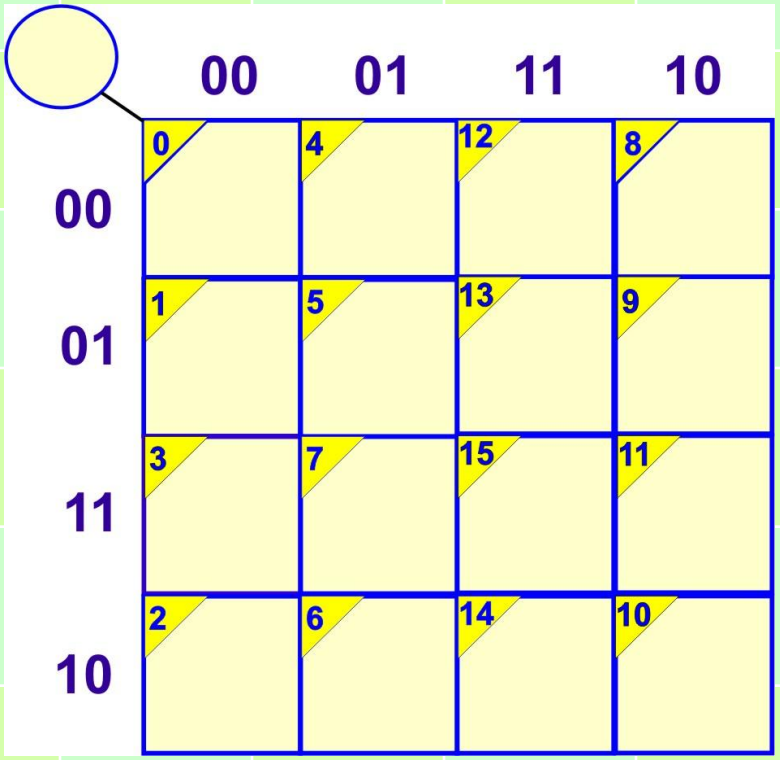
Tabla o mapa de Karnaugh, Kmap

Procedimiento gráfico para la simplificación de funciones algebraicas de un número de variables relativamente pequeño

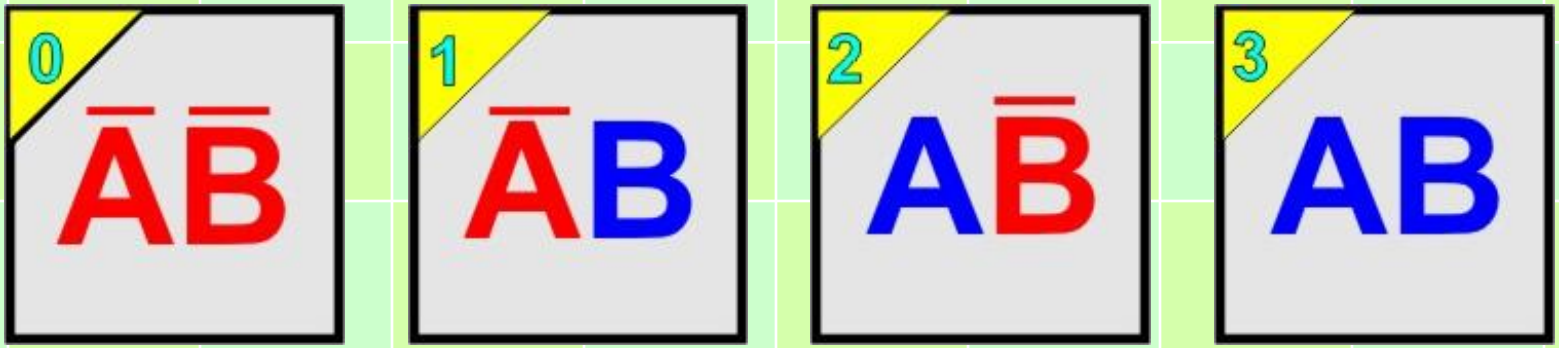
(en la práctica se puede utilizar para funciones de hasta seis variables).

Tabla o mapa de Karnaugh

Un diagrama o mapa de Karnaugh es una tabla de verdad dispuesta de manera adecuada para determinar por inspección la expresión mínima de suma de productos de una función lógica.



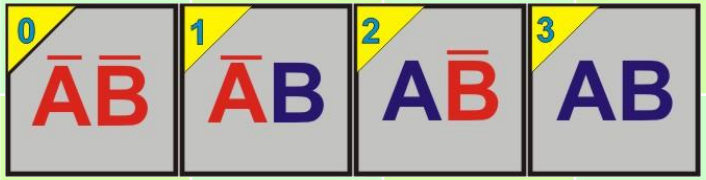
Con 2 variables A y B se pueden tener 4 Términos

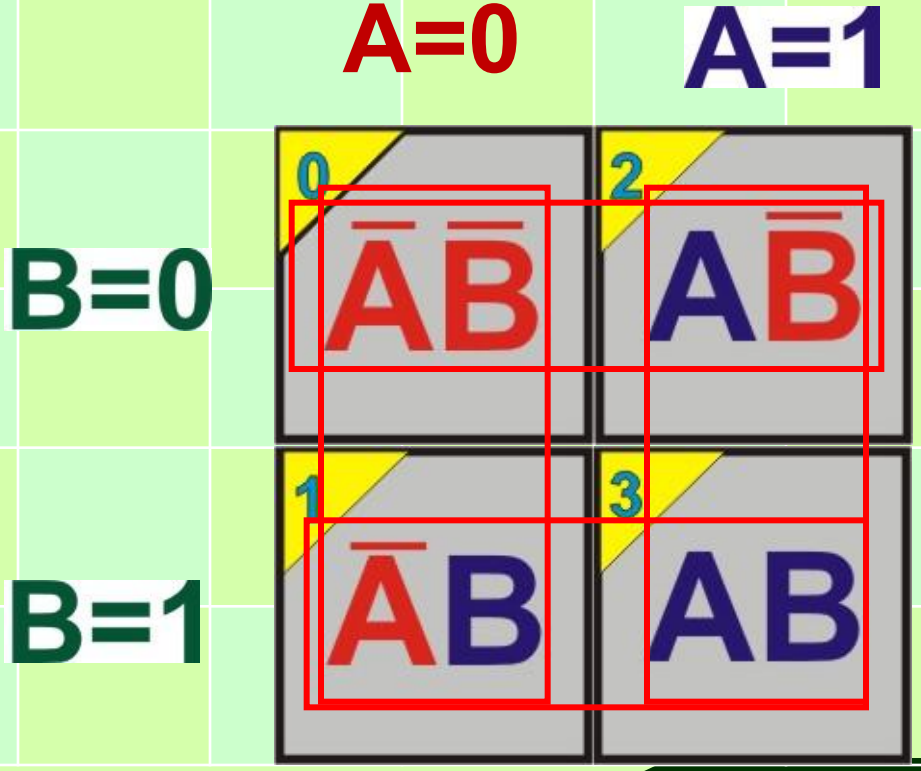


La factorización para la minimización se efectúa cuando solo cambia una variable entre dos términos semejantes y esta variable se elimina

Cada termino de dos variables tiene dos posibilidades de factorización

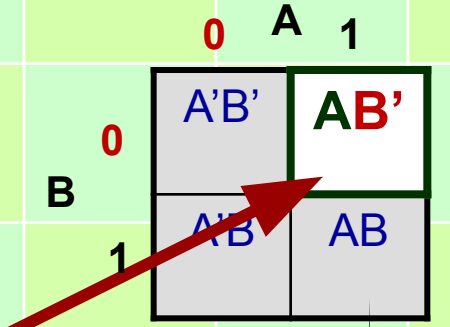
Kmap para 2 variables

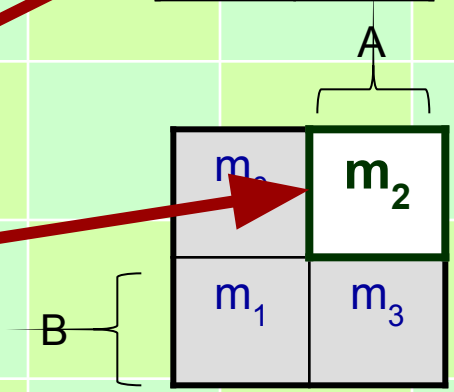


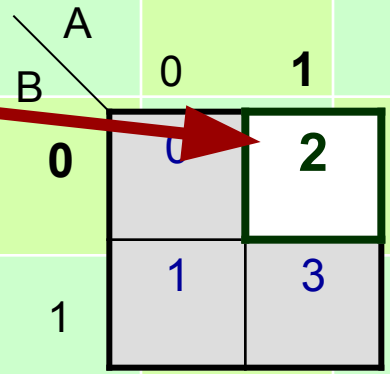


Mapa de Karnaugh para dos variables

m	A	B	S
0	0	0	
1	0	1	
2	1	0	AB’
3	1	1	

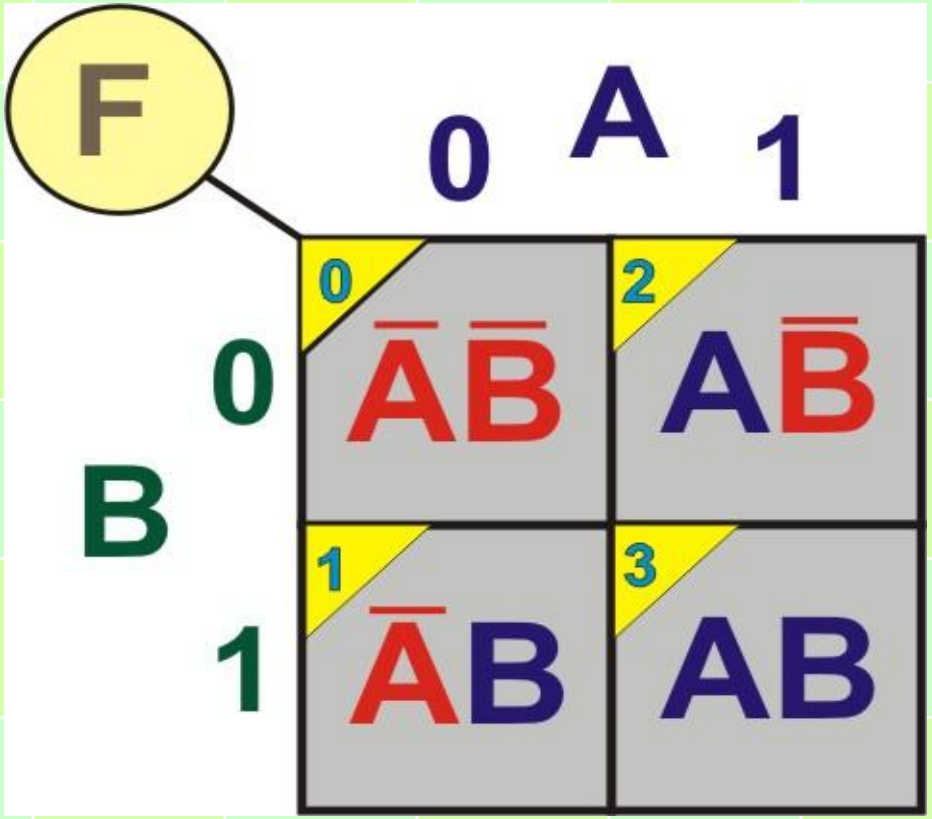




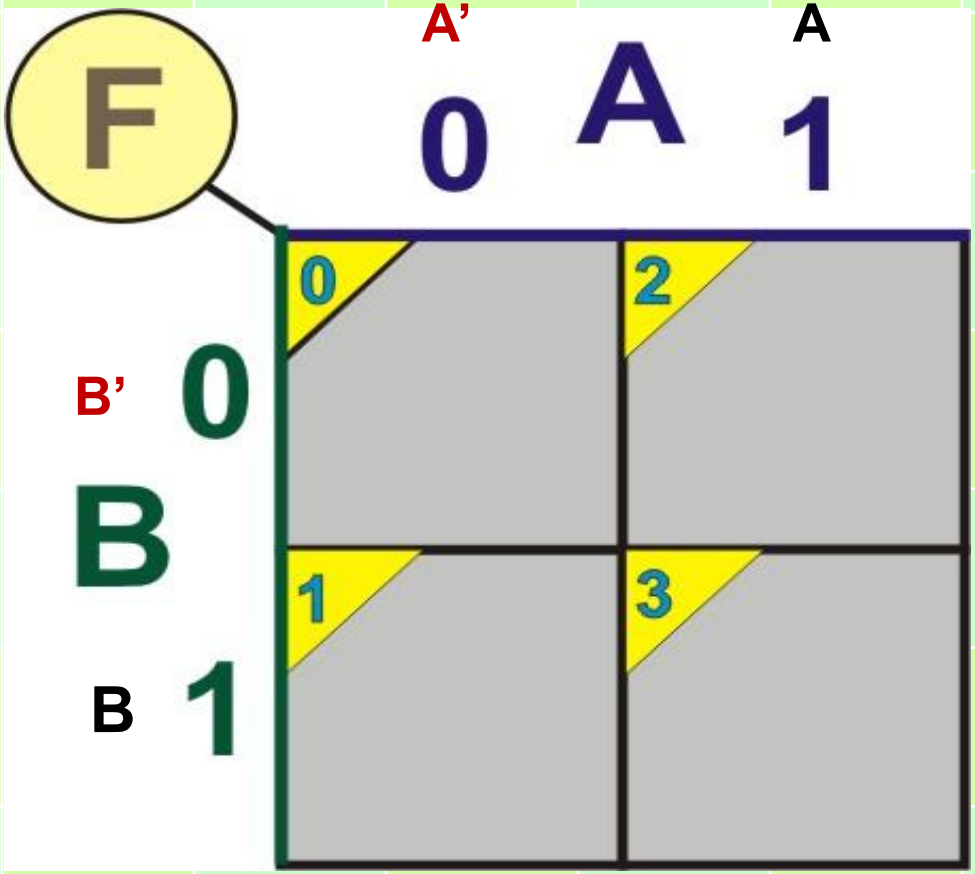


Kmap para 2 variables

0	1	2	3
$\bar{A}\bar{B}$	$\bar{A}B$	$A\bar{B}$	AB



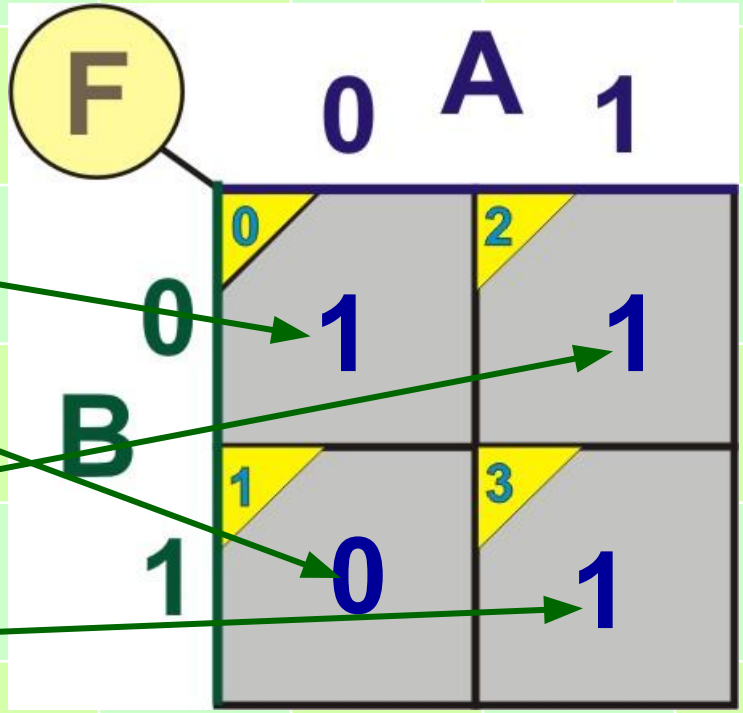
Kmap para 2 variables



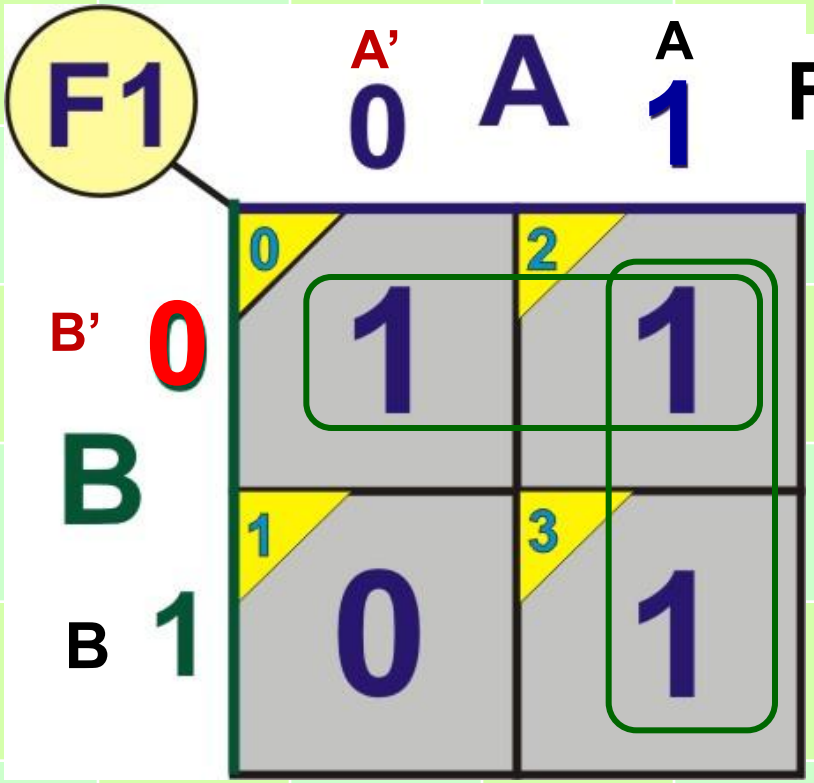
Como llenar el Kmap para 2 variables

$$F1_{(A,B)} = A'B' + AB' + AB$$

m	A	B	F1
0	0	0	1
1	0	1	0
2	1	0	1
3	1	1	1



Como resolver Kmap para 2 variables



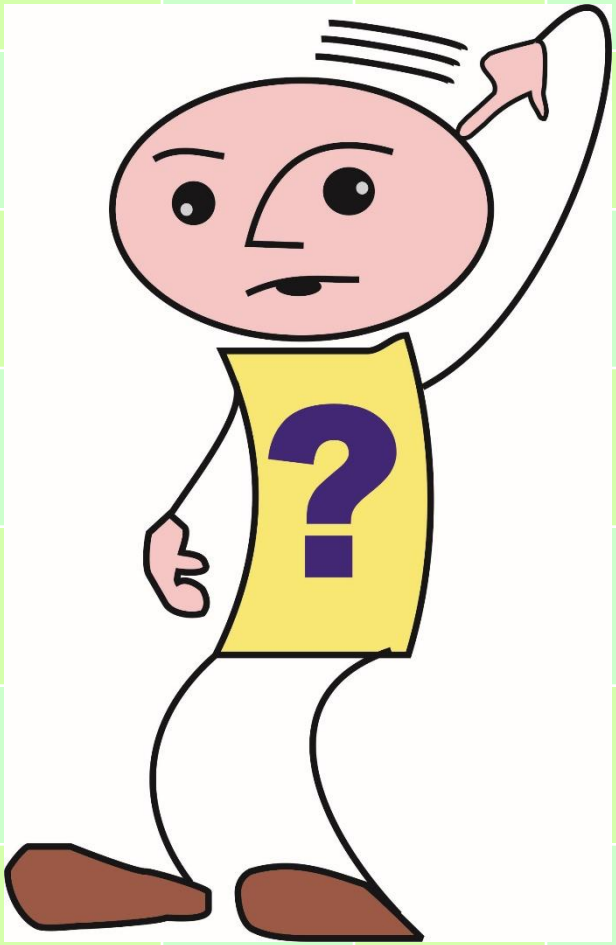
$$F1_{(A,B)} = A'B' + AB' + AB$$

$$F1_{(A,B)} = A + B'$$

Kmap para 3 variables

Con 3 Variables se tienen ___ términos

y cada termino tiene ___ posibilidades de factorización



Kmap para 3 variables

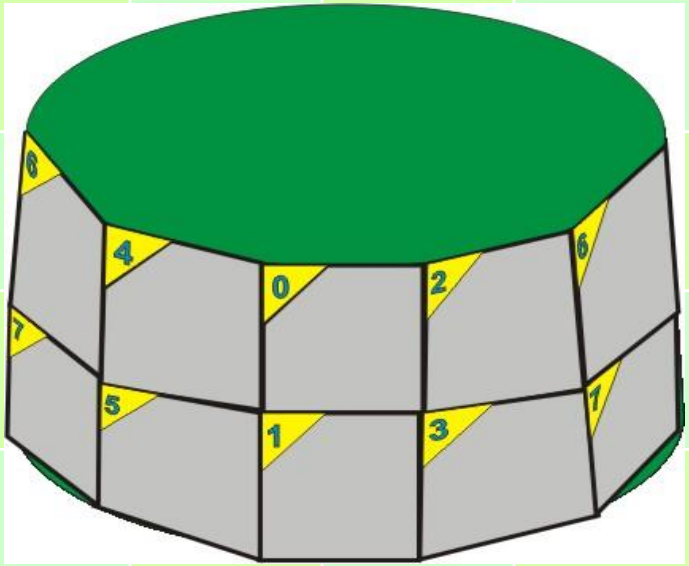
0 $\bar{A}\bar{B}\bar{C}$	1 $\bar{A}\bar{B}C$	2 $\bar{A}B\bar{C}$	3 $\bar{A}BC$
4 $A\bar{B}\bar{C}$	5 $A\bar{B}C$	6 $AB\bar{C}$	7 ABC

Cada termino tiene 3 posibilidades de factorización

4 $A\bar{B}\bar{C}$	0 $\bar{A}\bar{B}\bar{C}$	2 $\bar{A}B\bar{C}$	6 $AB\bar{C}$	4 $A\bar{B}\bar{C}$
5 $A\bar{B}C$	1 $\bar{A}\bar{B}C$	3 $\bar{A}BC$	7 ABC	5 $A\bar{B}C$

Kmap para 3 variables

4 $A\bar{B}\bar{C}$	0 $\bar{A}\bar{B}\bar{C}$	2 $\bar{A}B\bar{C}$	6 $AB\bar{C}$	4 $A\bar{B}\bar{C}$
5 $A\bar{B}C$	1 $\bar{A}\bar{B}C$	3 $\bar{A}BC$	7 ABC	5 $A\bar{B}C$



Mapa de Karnaugh para 3 variables

C \ AB	00	01	11	10
0	A'B'C'	A'BC'	ABC'	AB'C'
1	A'B'C	A'BC	ABC	AB'C

C \ AB	A'B' 00	A'B 01	AB 11	AB' 10
C' 0	0	2	6	4
C 1	1	3	7	5

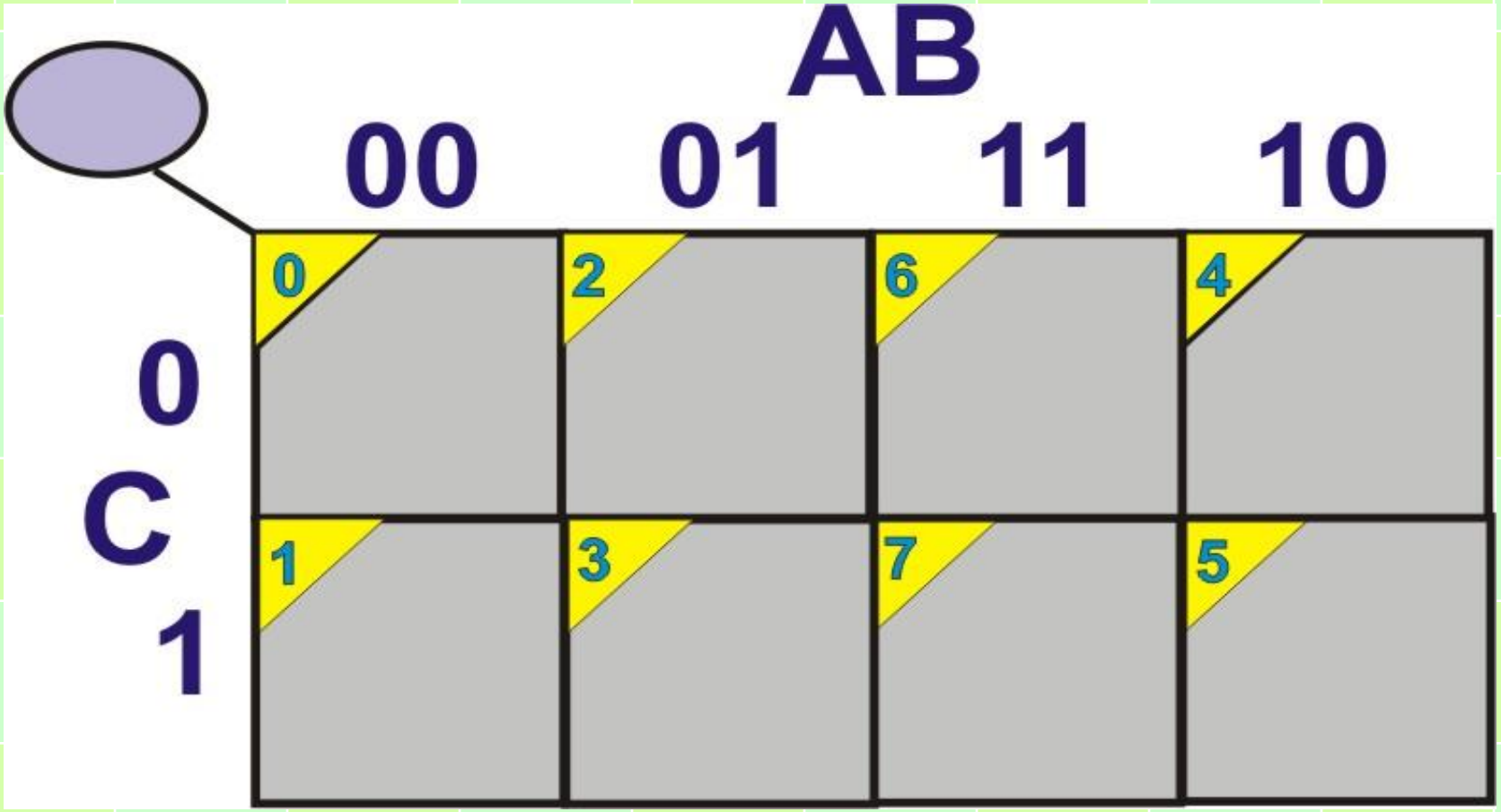
La idea con la codificación es poder usar el P9a. ab+ab'=a

Kmap para 3 variables

4 $A\bar{B}\bar{C}$	0 $\bar{A}\bar{B}\bar{C}$	2 $\bar{A}B\bar{C}$	6 $AB\bar{C}$	4 $A\bar{B}\bar{C}$
5 $A\bar{B}C$	1 $\bar{A}\bar{B}C$	3 $\bar{A}BC$	7 ABC	5 $A\bar{B}C$

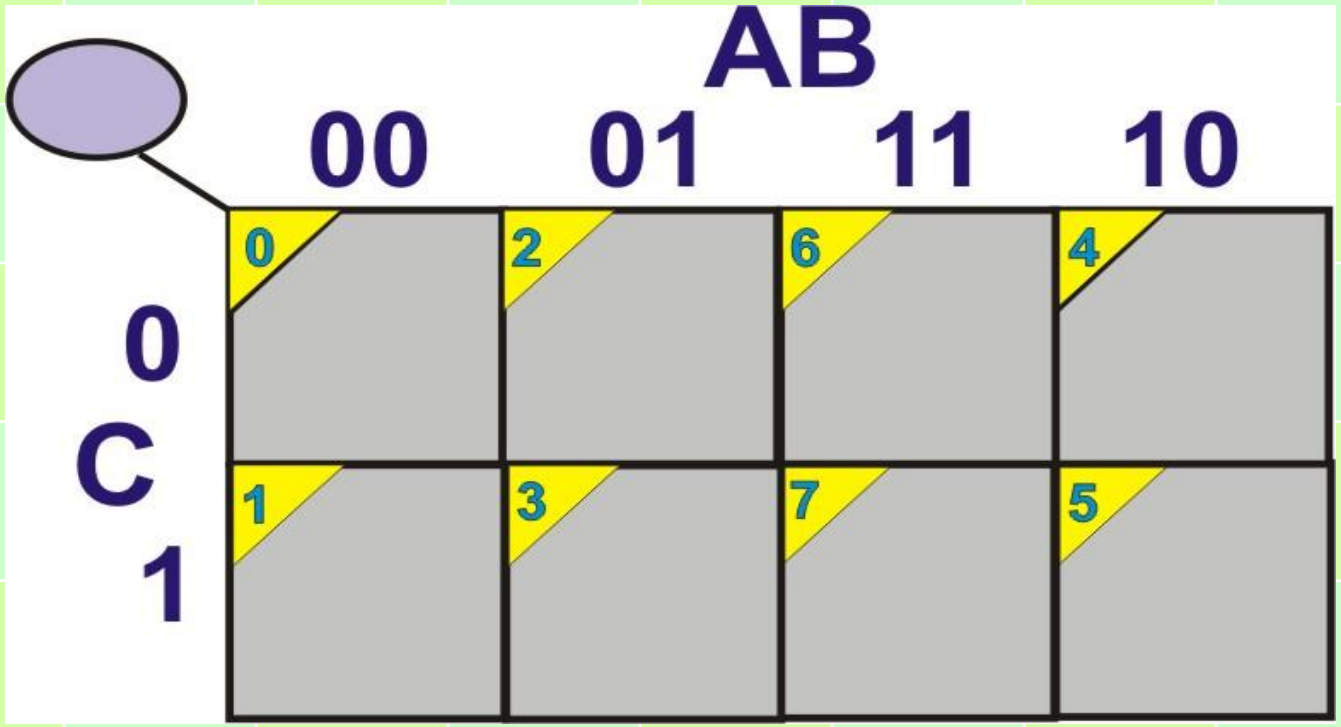
C \ AB	00	01	11	10
0	0 $\bar{A}\bar{B}\bar{C}$	2 $\bar{A}B\bar{C}$	6 $AB\bar{C}$	4 $A\bar{B}\bar{C}$
1	1 $\bar{A}\bar{B}C$	3 $\bar{A}BC$	7 ABC	5 $A\bar{B}C$

Kmap para 3 variables



Kmap para 3 variables

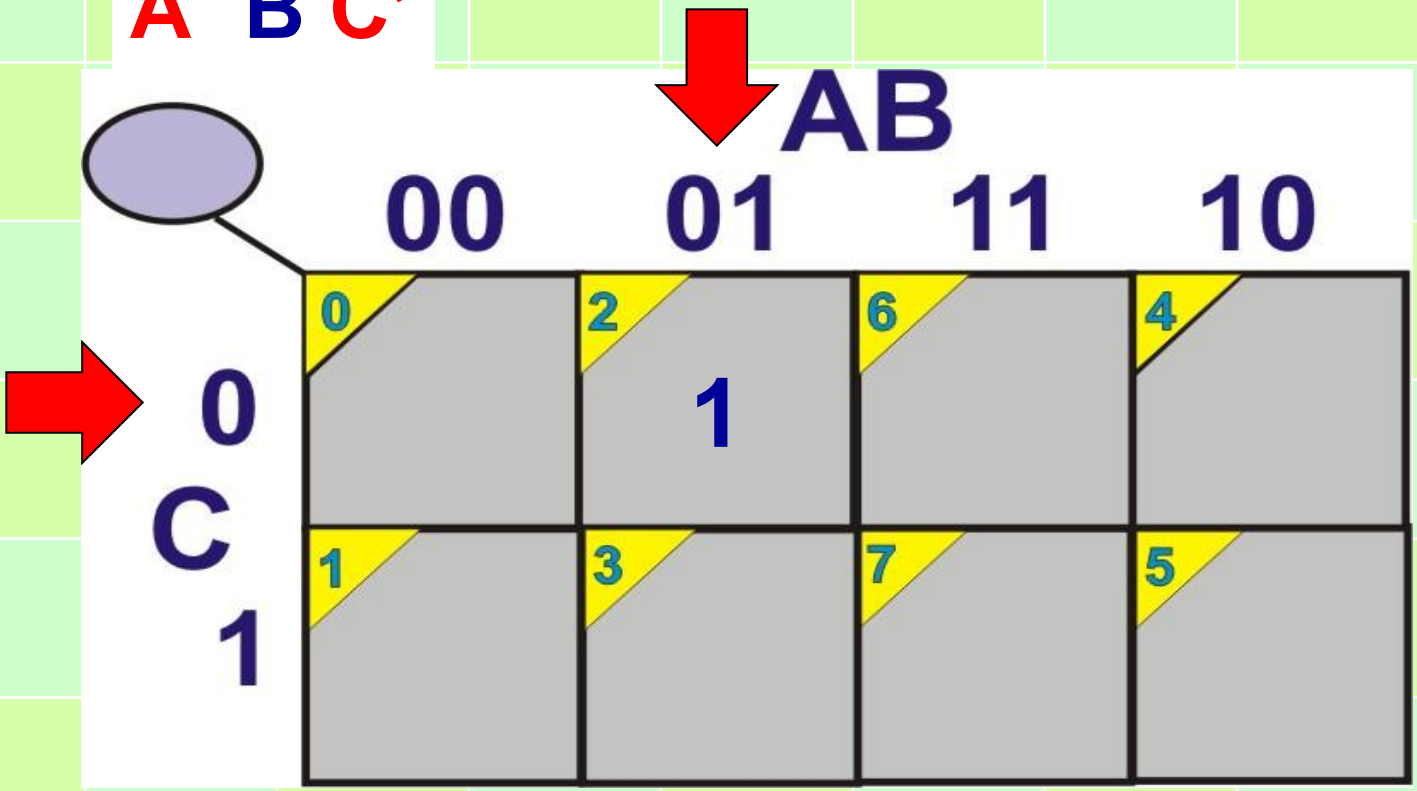
$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$



Kmap para 3 variables

$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

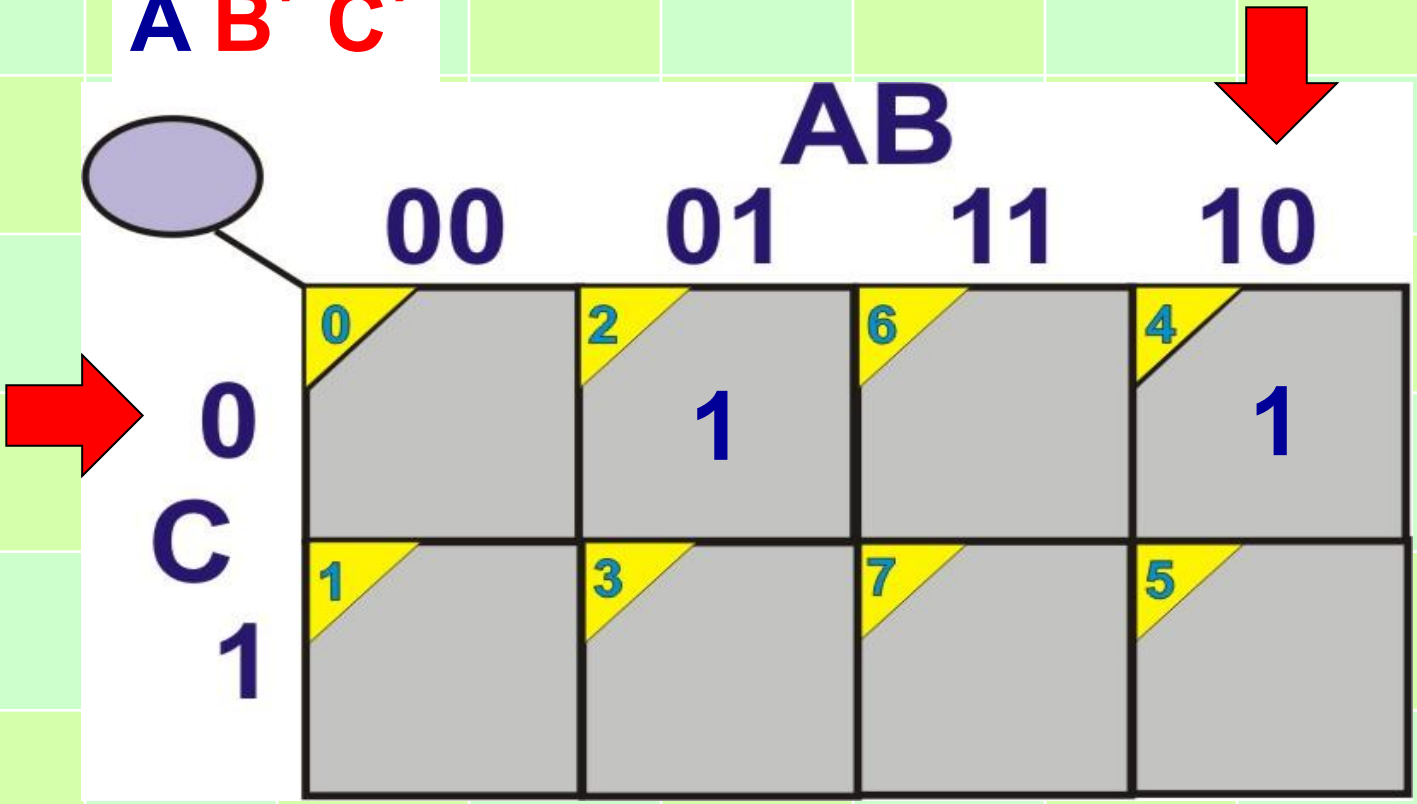
A´ B C’



Kmap para 3 variables

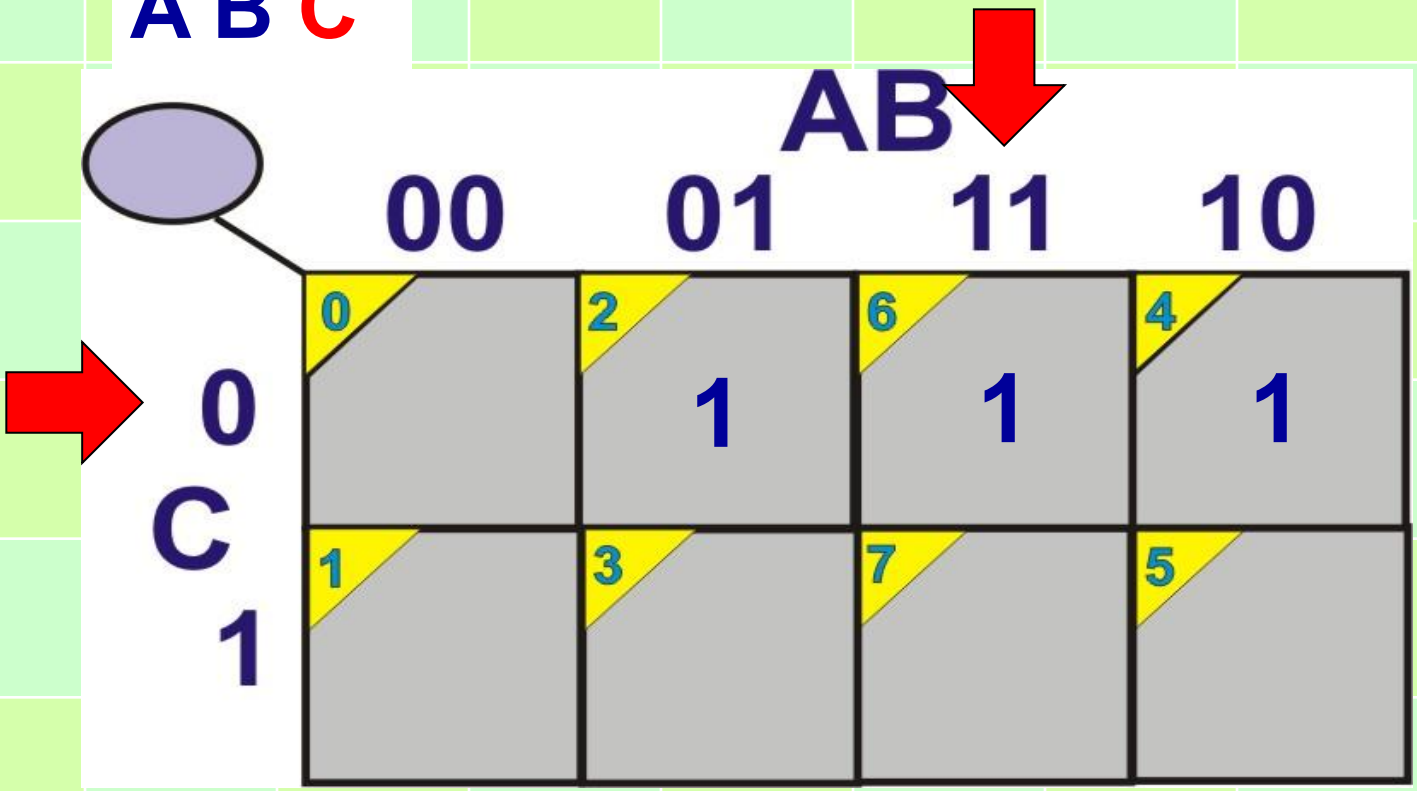
$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

$A B' C'$



$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

A B C´



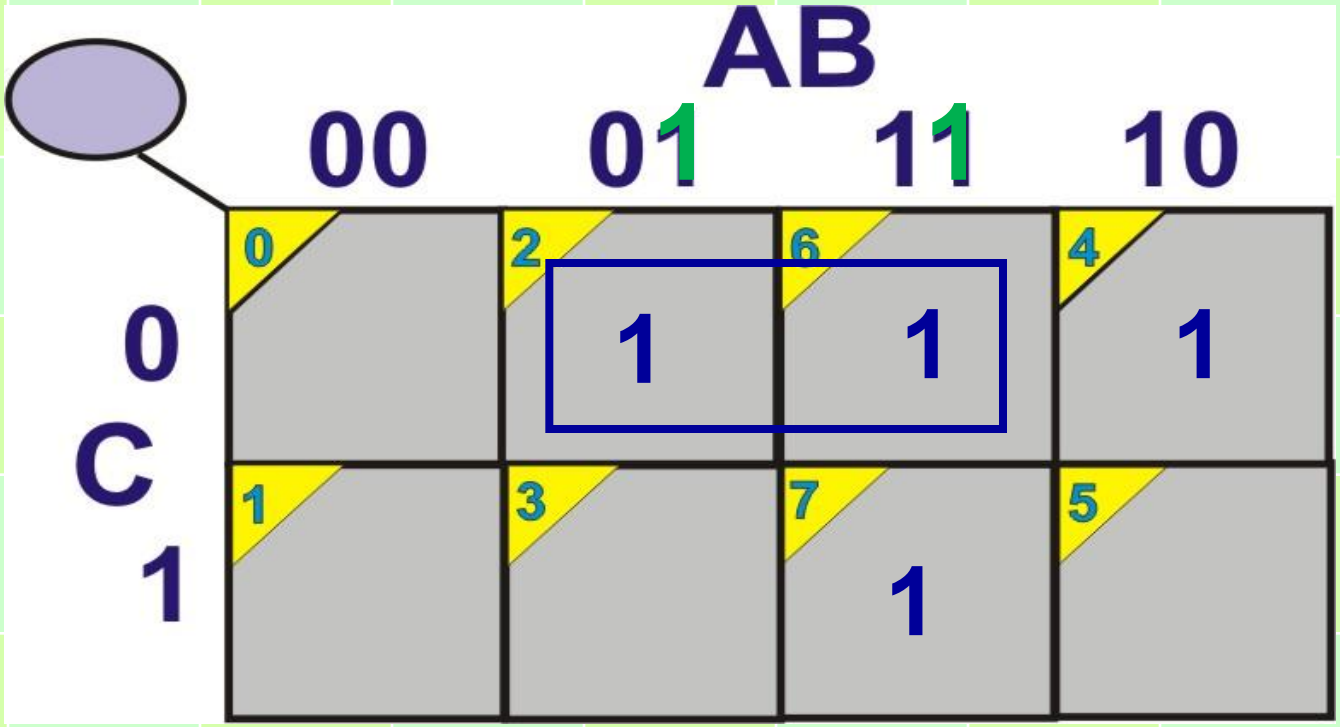
$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

A B C

C \ AB	00	01	11	10
0	0	2: 1	6: 1	4: 1
1	1	3	7: 1	5

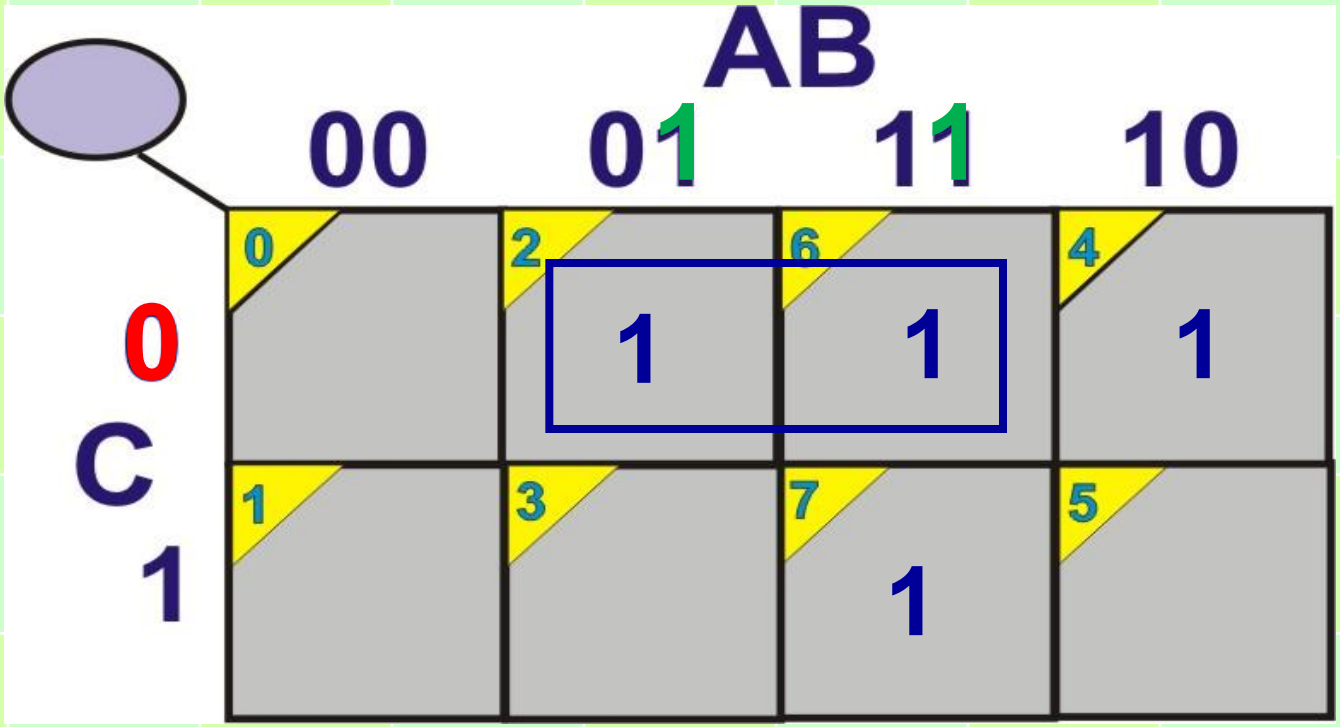
$$F_{(A, B, C)} = \boxed{A' B C'} + A B' C' + \boxed{A B C'} + A B C$$

$$F_{(A, B, C)} = B$$



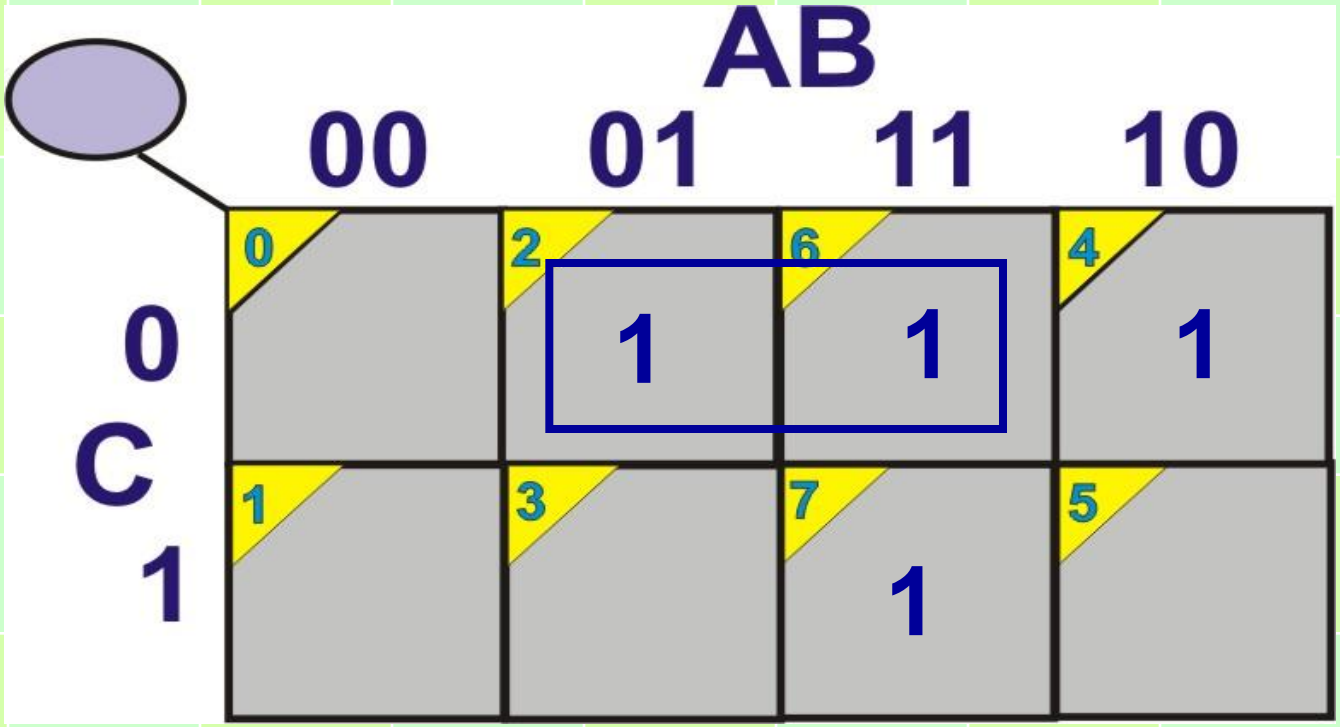
$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

$$F_{(A, B, C)} = B C' +$$



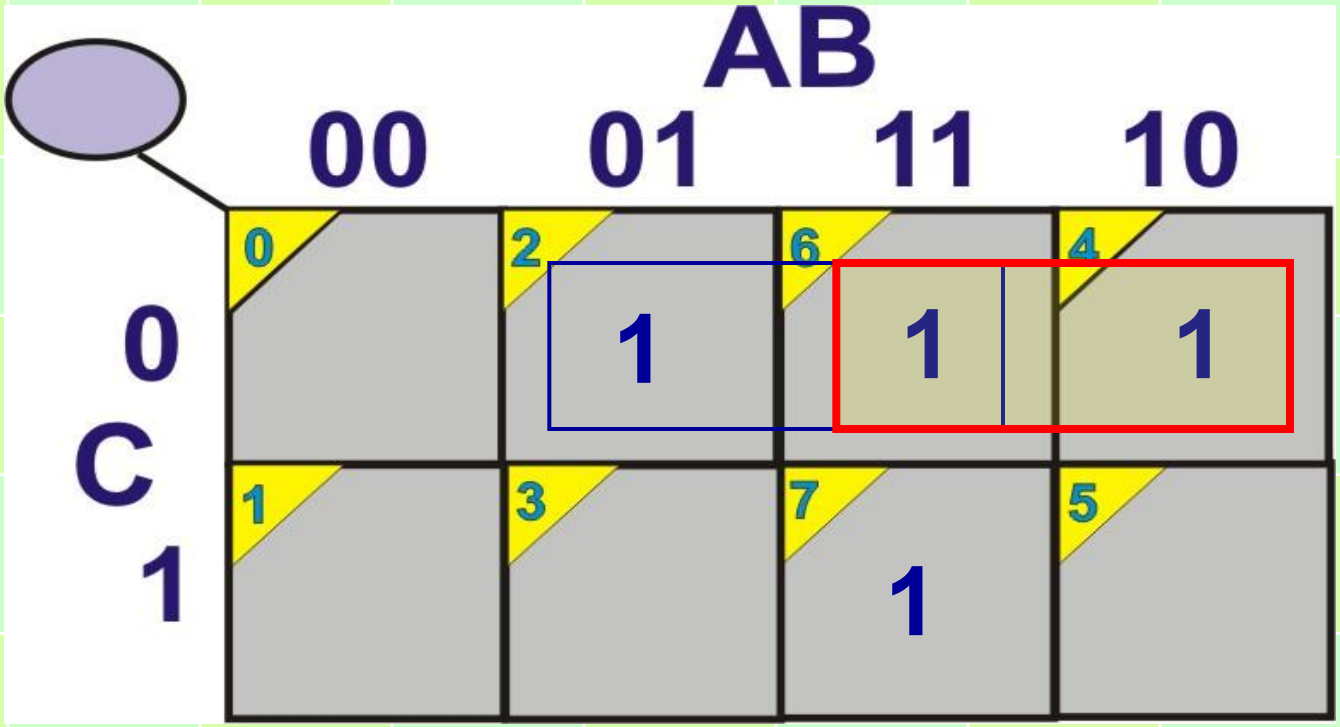
$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

$$F_{(A, B, C)} = B C' +$$



$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

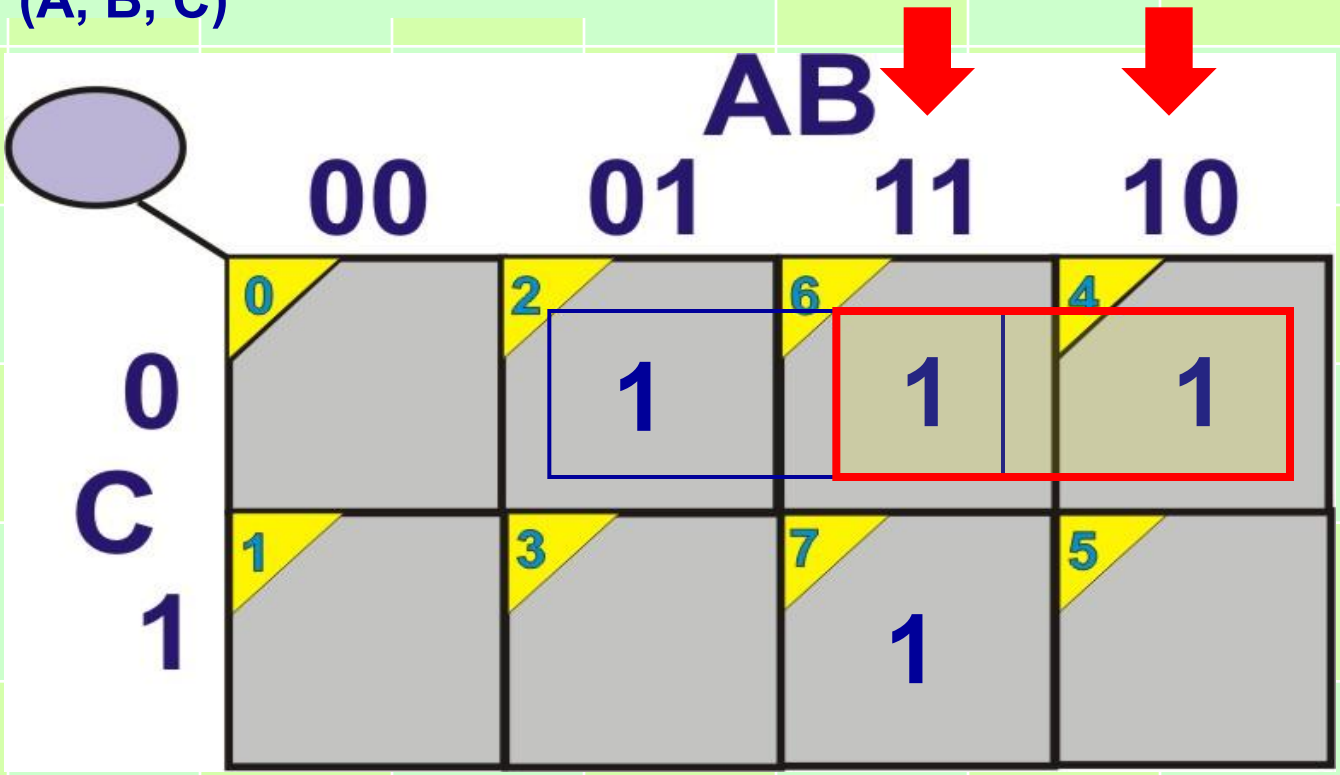
$$F_{(A, B, C)} = B C' +$$



Kmap para 3 variables

$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

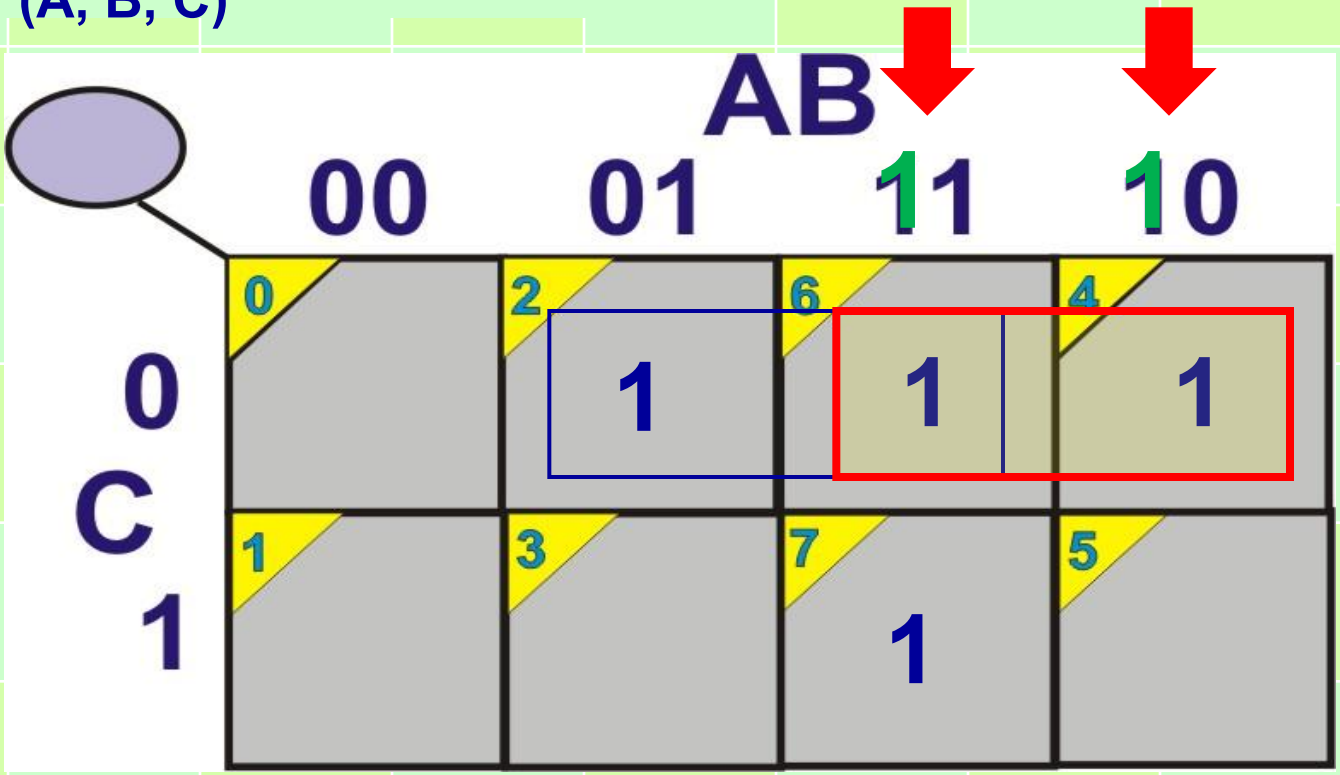
$$F_{(A, B, C)} = B C' +$$



Kmap para 3 variables

$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

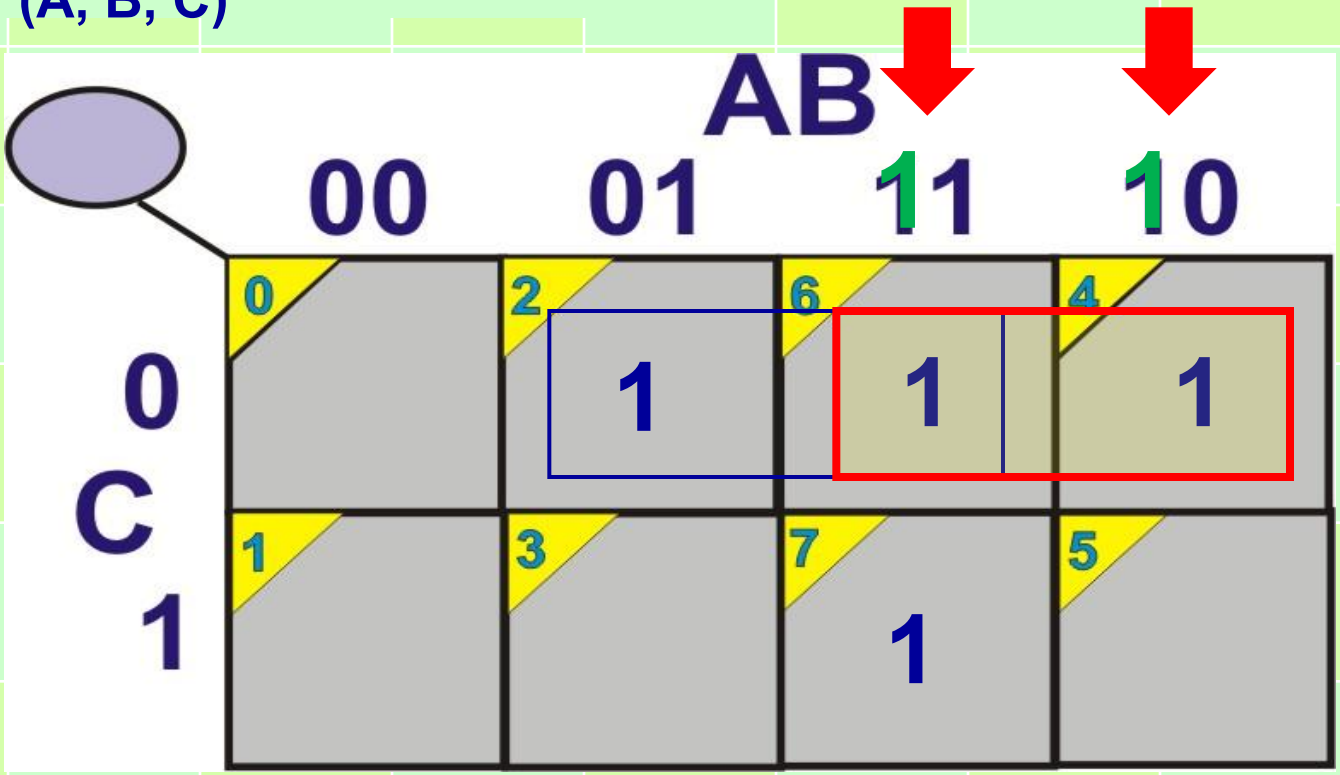
$$F_{(A, B, C)} = B C' +$$



Kmap para 3 variables

$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

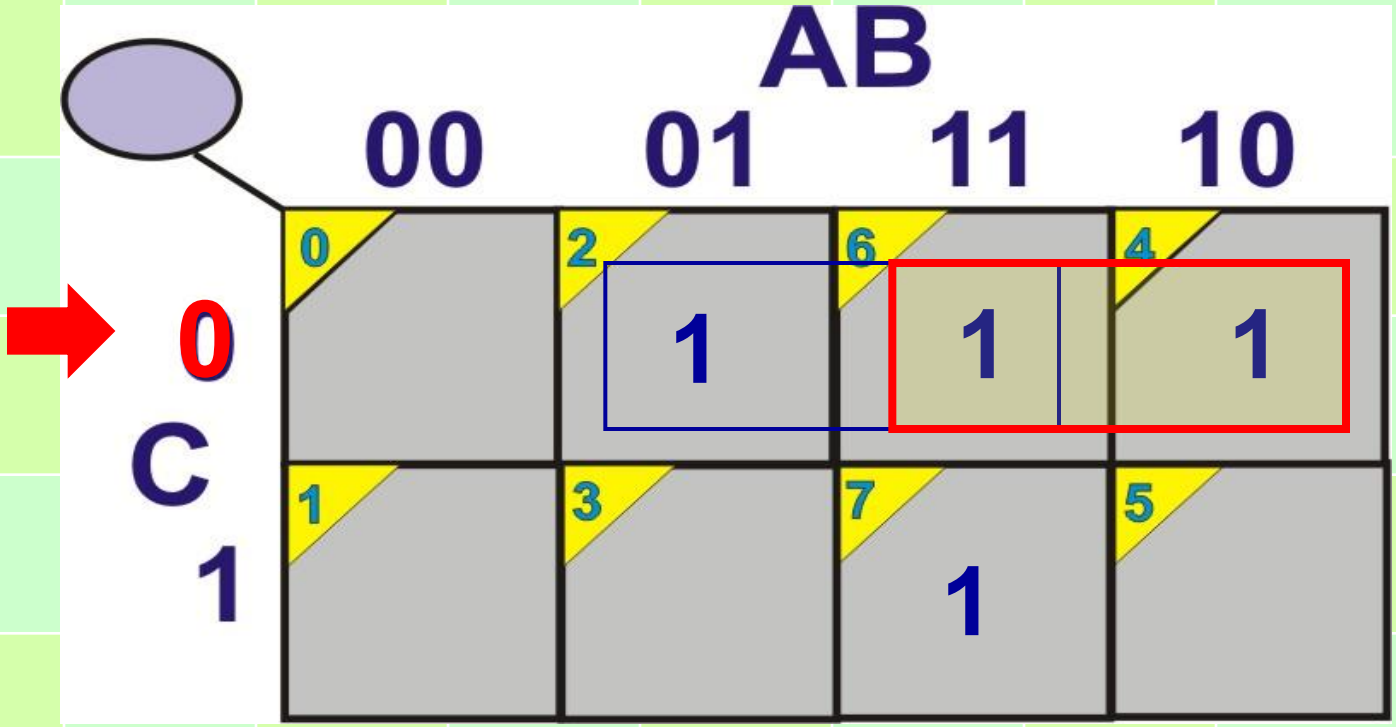
$$F_{(A, B, C)} = B C' + A$$



Kmap para 3 variables

$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

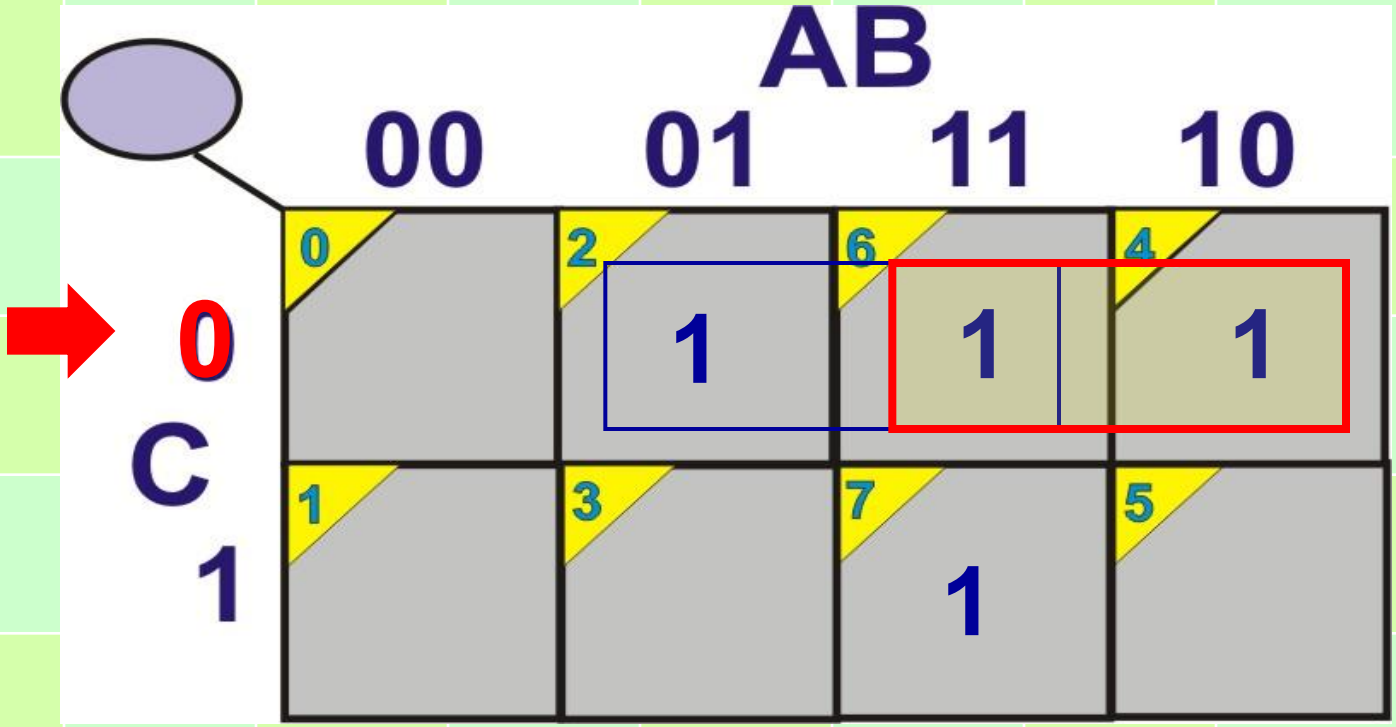
$$F_{(A, B, C)} = B C' + A$$



Kmap para 3 variables

$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

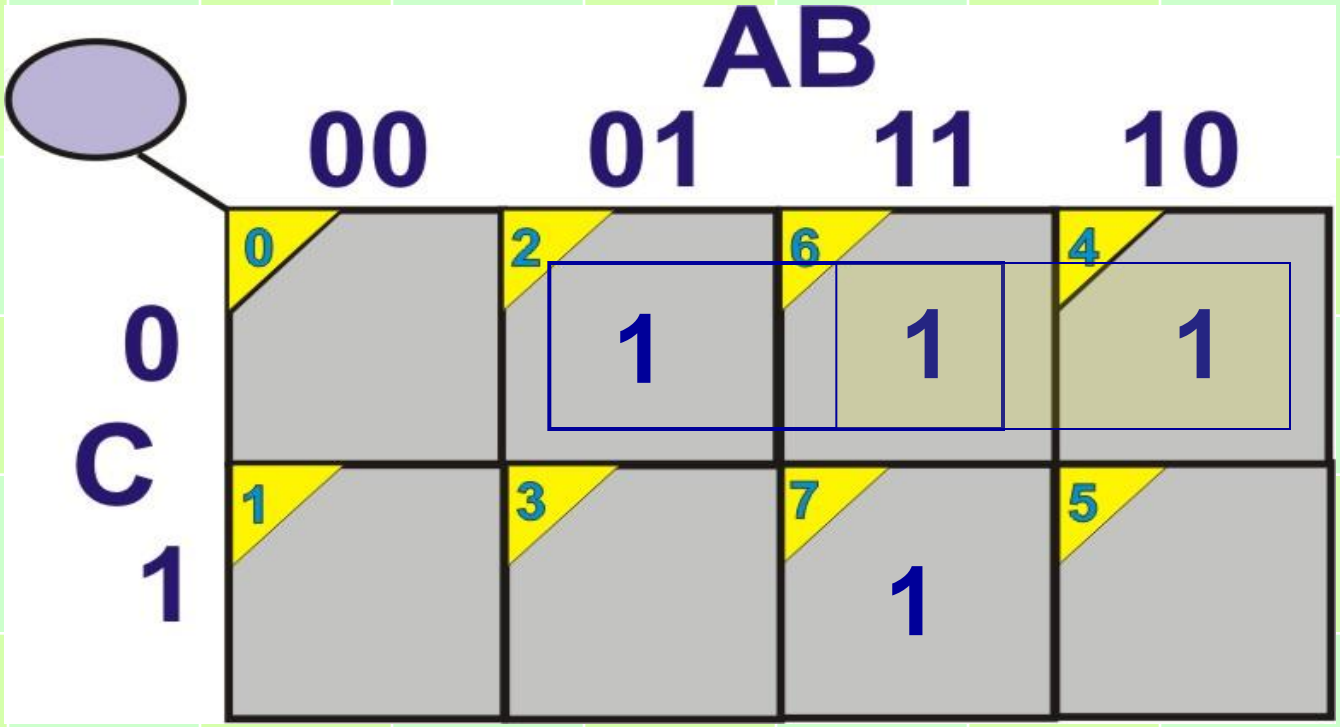
$$F_{(A, B, C)} = B C' + A C'$$



Kmap para 3 variables

$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

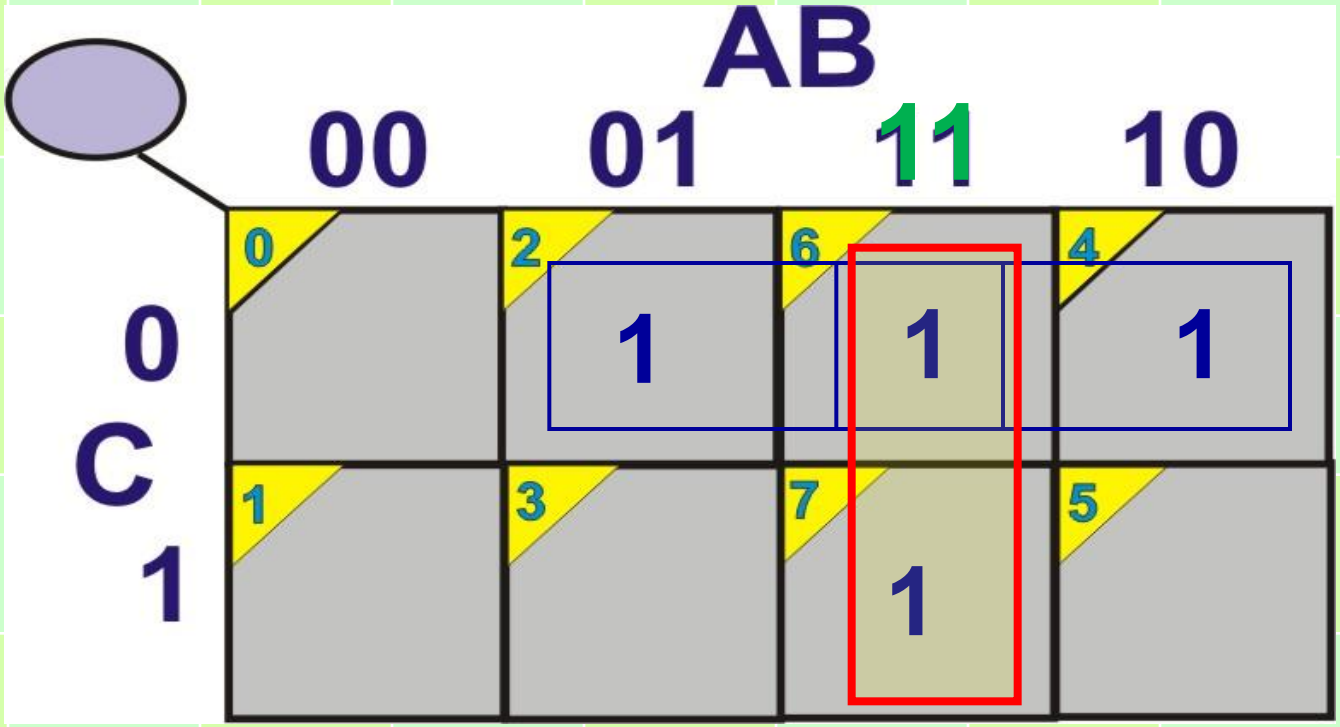
$$F_{(A, B, C)} = B C' + A C' +$$



Kmap para 3 variables

$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

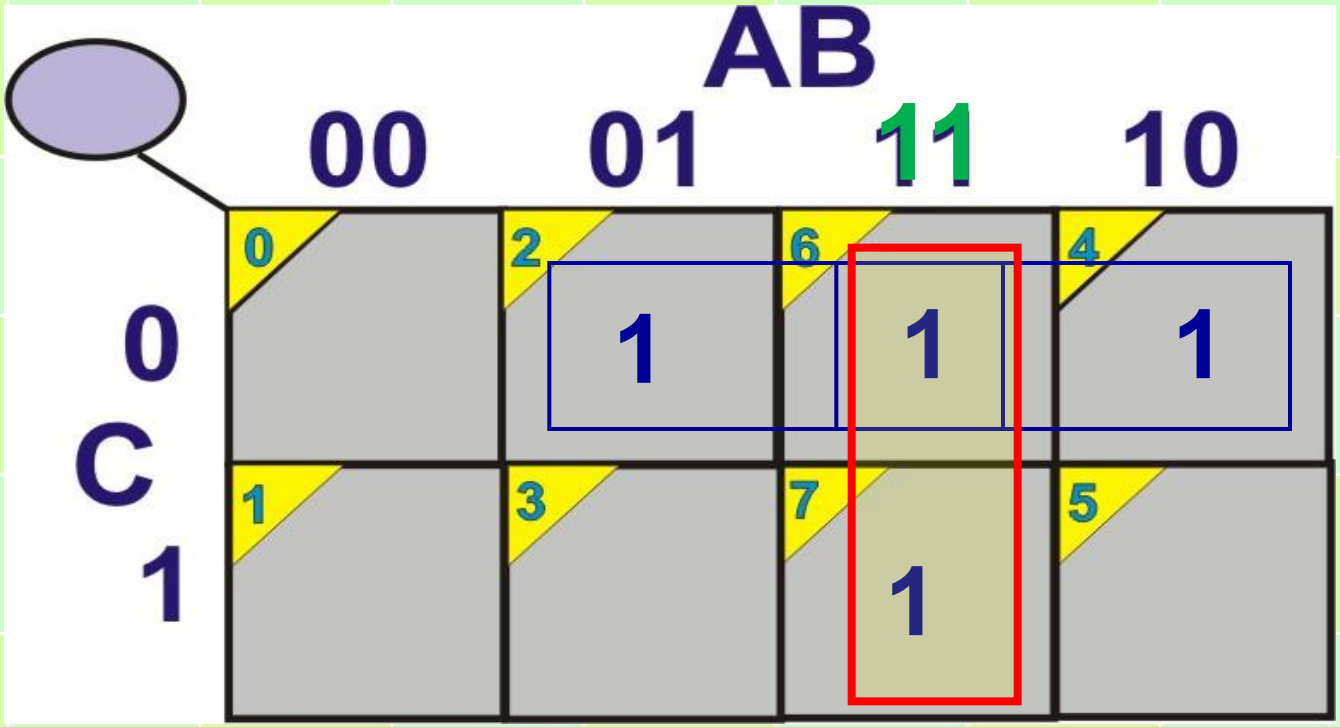
$$F_{(A, B, C)} = B C' + A C' +$$



Kmap para 3 variables

$$F_{(A, B, C)} = A' B C' + A B' C' + A B C' + A B C$$

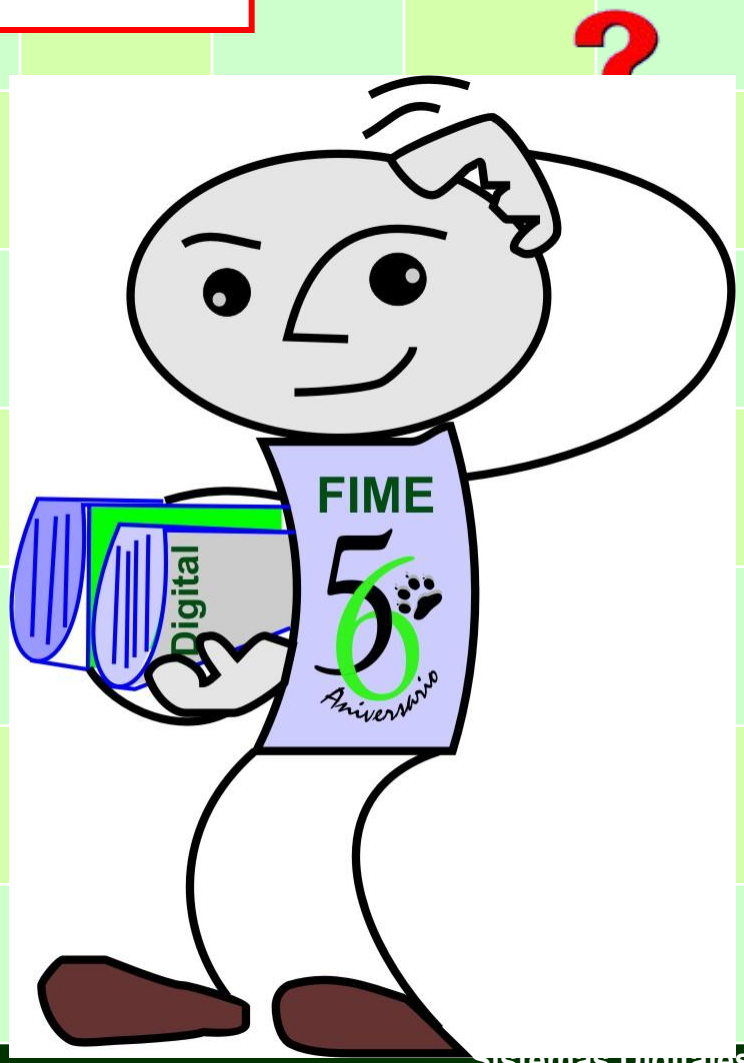
$$F_{(A, B, C)} = B C' + A C' + A B$$



Kmap para 4 variables

Con 4 Variables se tienen __ términos

y cada termino tiene __ posibilidades de factorización



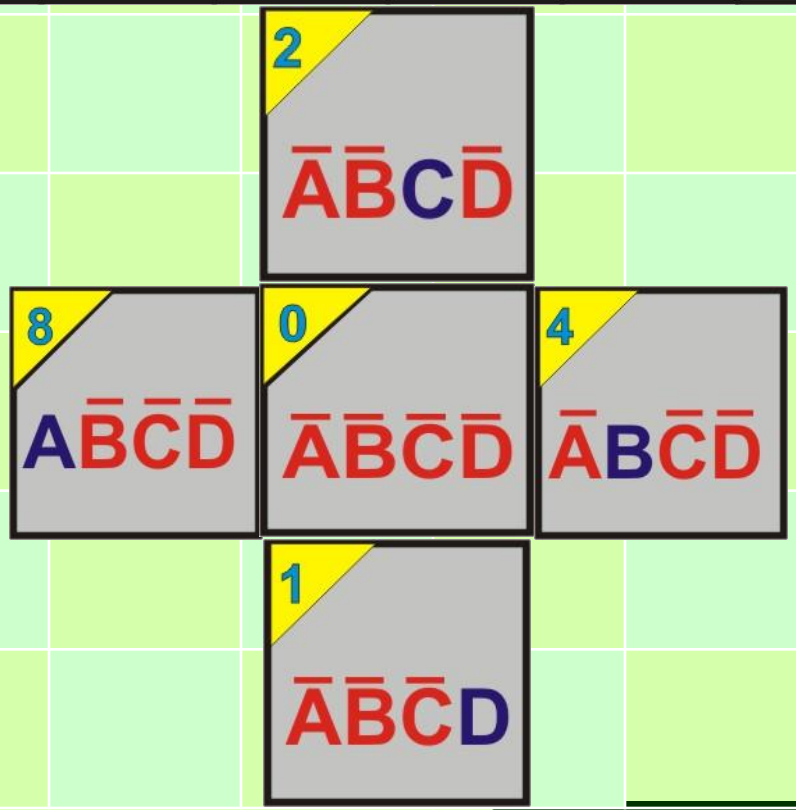
Kmap para 4 variables

0 $\bar{A}\bar{B}\bar{C}\bar{D}$	1 $\bar{A}\bar{B}\bar{C}D$	2 $\bar{A}\bar{B}C\bar{D}$	3 $\bar{A}\bar{B}CD$	4 $\bar{A}B\bar{C}\bar{D}$	5 $\bar{A}B\bar{C}D$	6 $\bar{A}BC\bar{D}$	7 $\bar{A}BCD$
8 $A\bar{B}\bar{C}\bar{D}$	9 $A\bar{B}\bar{C}D$	10 $A\bar{B}C\bar{D}$	11 $A\bar{B}CD$	12 $AB\bar{C}\bar{D}$	13 $AB\bar{C}D$	14 $ABC\bar{D}$	15 $ABCD$

Cada termino tiene 4 posibilidades de factorización

Cada termino tiene 4 posibilidades de factorización

0 $\bar{A}\bar{B}\bar{C}\bar{D}$	1 $\bar{A}\bar{B}\bar{C}D$	2 $\bar{A}\bar{B}C\bar{D}$	3 $\bar{A}\bar{B}CD$	4 $\bar{A}B\bar{C}\bar{D}$	5 $\bar{A}B\bar{C}D$	6 $\bar{A}BC\bar{D}$	7 $\bar{A}BCD$
8 $A\bar{B}\bar{C}\bar{D}$	9 $A\bar{B}\bar{C}D$	10 $A\bar{B}C\bar{D}$	11 $A\bar{B}CD$	12 $AB\bar{C}\bar{D}$	13 $A\bar{B}\bar{C}D$	14 $ABC\bar{D}$	15 $ABCD$



K map para 4 variables

10 $A\bar{B}C\bar{D}$	2 $\bar{A}\bar{B}C\bar{D}$	6 $\bar{A}BC\bar{D}$	14 $ABC\bar{D}$	10 $A\bar{B}C\bar{D}$
8 $A\bar{B}\bar{C}\bar{D}$	0 $\bar{A}\bar{B}\bar{C}\bar{D}$	4 $\bar{A}B\bar{C}\bar{D}$	12 $AB\bar{C}\bar{D}$	8 $A\bar{B}\bar{C}\bar{D}$
9 $A\bar{B}\bar{C}D$	1 $\bar{A}\bar{B}\bar{C}D$	5 $\bar{A}B\bar{C}D$	13 $A\bar{B}\bar{C}D$	9 $A\bar{B}\bar{C}D$
11 $A\bar{B}CD$	3 $\bar{A}\bar{B}CD$	7 $\bar{A}BCD$	15 $ABCD$	11 $A\bar{B}CD$
10 $A\bar{B}C\bar{D}$	2 $\bar{A}\bar{B}C\bar{D}$	6 $\bar{A}BC\bar{D}$	14 $ABC\bar{D}$	10 $A\bar{B}C\bar{D}$

K map para 4 variables

AB →

10	00	01	11	10
10 $A\bar{B}C\bar{D}$	2 $\bar{A}\bar{B}C\bar{D}$	6 $\bar{A}BC\bar{D}$	14 $ABC\bar{D}$	10 $A\bar{B}C\bar{D}$
8 $A\bar{B}\bar{C}\bar{D}$	0 $\bar{A}\bar{B}\bar{C}\bar{D}$	4 $\bar{A}B\bar{C}\bar{D}$	12 $AB\bar{C}\bar{D}$	8 $A\bar{B}\bar{C}\bar{D}$
9 $A\bar{B}\bar{C}D$	1 $\bar{A}\bar{B}\bar{C}D$	5 $\bar{A}B\bar{C}D$	13 $A\bar{B}\bar{C}D$	9 $A\bar{B}\bar{C}D$
11 $A\bar{B}CD$	3 $\bar{A}\bar{B}CD$	7 $\bar{A}BCD$	15 $ABCD$	11 $A\bar{B}CD$
10 $A\bar{B}C\bar{D}$	2 $\bar{A}\bar{B}C\bar{D}$	6 $\bar{A}BC\bar{D}$	14 $ABC\bar{D}$	10 $A\bar{B}C\bar{D}$

Kmap para 4 variables

CD \ AB	10	00	01	11	10
10	10 $A\bar{B}C\bar{D}$	2 $\bar{A}\bar{B}C\bar{D}$	6 $\bar{A}BC\bar{D}$	14 $ABC\bar{D}$	10 $A\bar{B}C\bar{D}$
00	8 $A\bar{B}\bar{C}\bar{D}$	0 $\bar{A}\bar{B}\bar{C}\bar{D}$	4 $\bar{A}B\bar{C}\bar{D}$	12 $AB\bar{C}\bar{D}$	8 $A\bar{B}\bar{C}\bar{D}$
01	9 $A\bar{B}\bar{C}D$	1 $\bar{A}\bar{B}\bar{C}D$	5 $\bar{A}B\bar{C}D$	13 $A\bar{B}\bar{C}D$	9 $A\bar{B}\bar{C}D$
11	11 $A\bar{B}CD$	3 $\bar{A}\bar{B}CD$	7 $\bar{A}BCD$	15 $ABCD$	11 $A\bar{B}CD$
10	10 $A\bar{B}C\bar{D}$	2 $\bar{A}\bar{B}C\bar{D}$	6 $\bar{A}BC\bar{D}$	14 $ABC\bar{D}$	10 $A\bar{B}C\bar{D}$

Kmap para 4 variables

CD \ AB	00	01	11	10
00	0 $\bar{A}\bar{B}\bar{C}\bar{D}$	4 $\bar{A}B\bar{C}\bar{D}$	12 $AB\bar{C}\bar{D}$	8 $A\bar{B}\bar{C}\bar{D}$
01	1 $\bar{A}\bar{B}\bar{C}D$	5 $\bar{A}B\bar{C}D$	13 $AB\bar{C}D$	9 $A\bar{B}\bar{C}D$
11	3 $\bar{A}\bar{B}CD$	7 $\bar{A}BCD$	15 $ABCD$	11 $A\bar{B}CD$
10	2 $\bar{A}\bar{B}C\bar{D}$	6 $\bar{A}BC\bar{D}$	14 $ABC\bar{D}$	10 $A\bar{B}C\bar{D}$

10 $A\bar{B}C\bar{D}$	2 $\bar{A}\bar{B}C\bar{D}$
8 $A\bar{B}\bar{C}\bar{D}$	0 $\bar{A}\bar{B}\bar{C}\bar{D}$

Mapa de Karnaugh para 4 variables

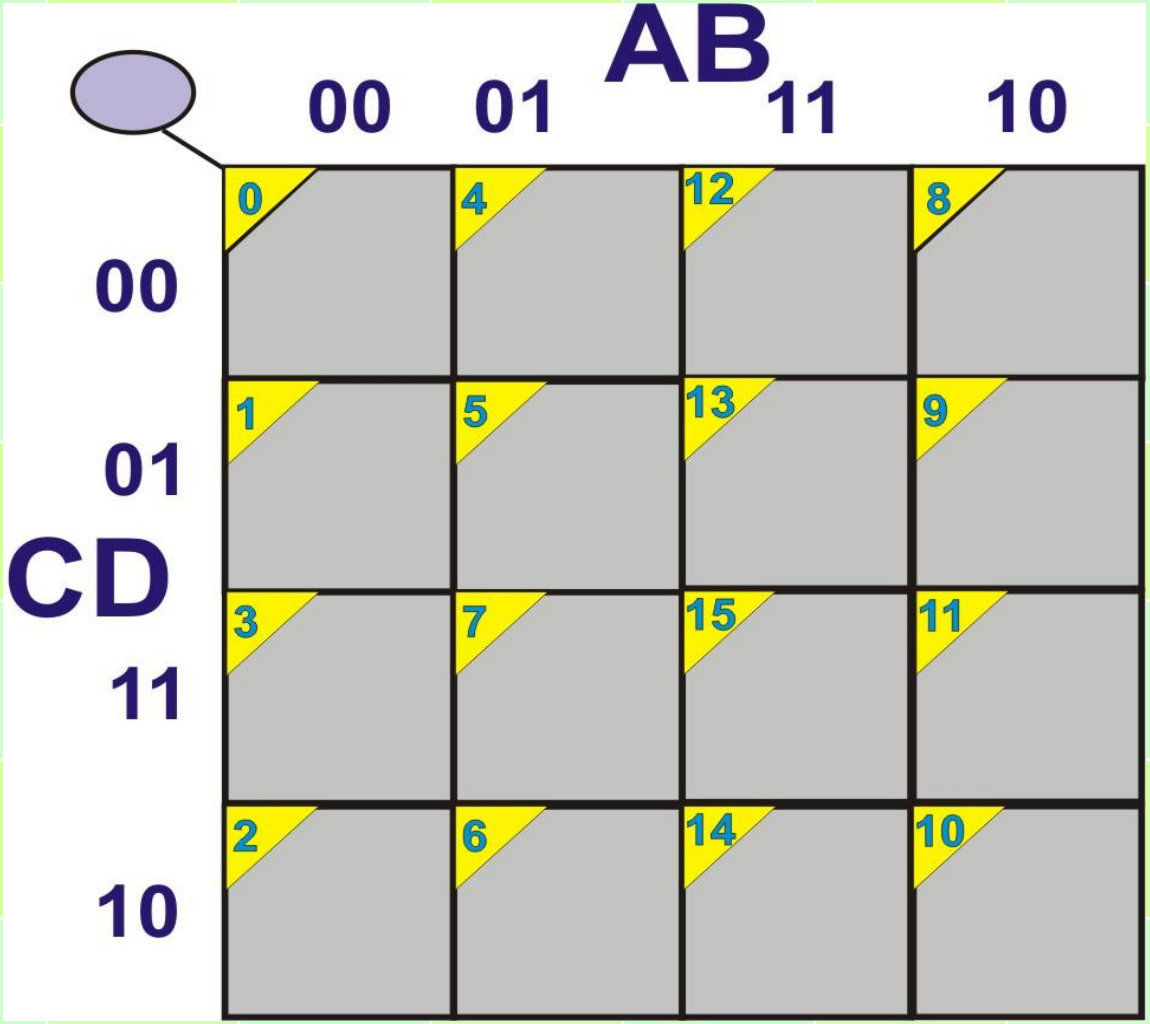
CD \ AB	00	01	11	10
00	A’B’C’D’	A’BC’D’	ABC’D’	AB’C’D’
01	A’B’C’D	A’BC’D	ABC’D	AB’C’D
11	A’B’CD	A’BCD	ABCD	AB’CD
10	A’B’CD’	A’BCD’	ABCD’	AB’CD’

AB

CD \ AB	A’B’ 00	A’B 01	AB 11	AB’ 10
C’D’ 00	0	4	12	8
C’D 01	1	5	13	9
CD 11	3	7	15	11
CD’ 10	2	6	14	10

CD

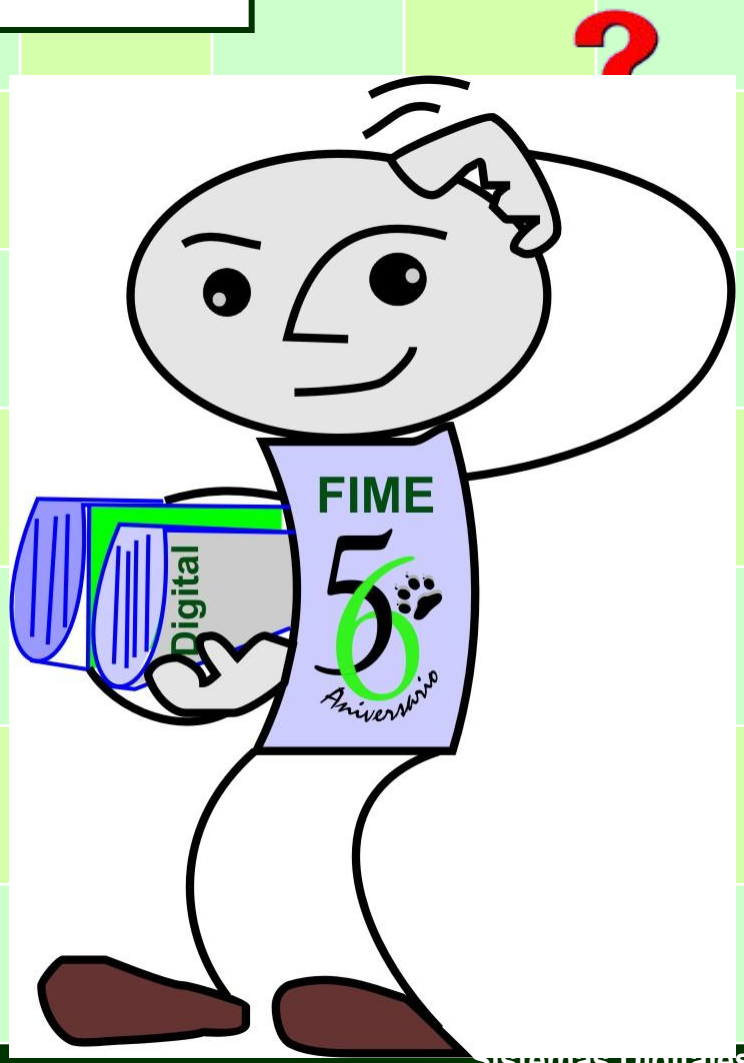
Kmap para 4 variables



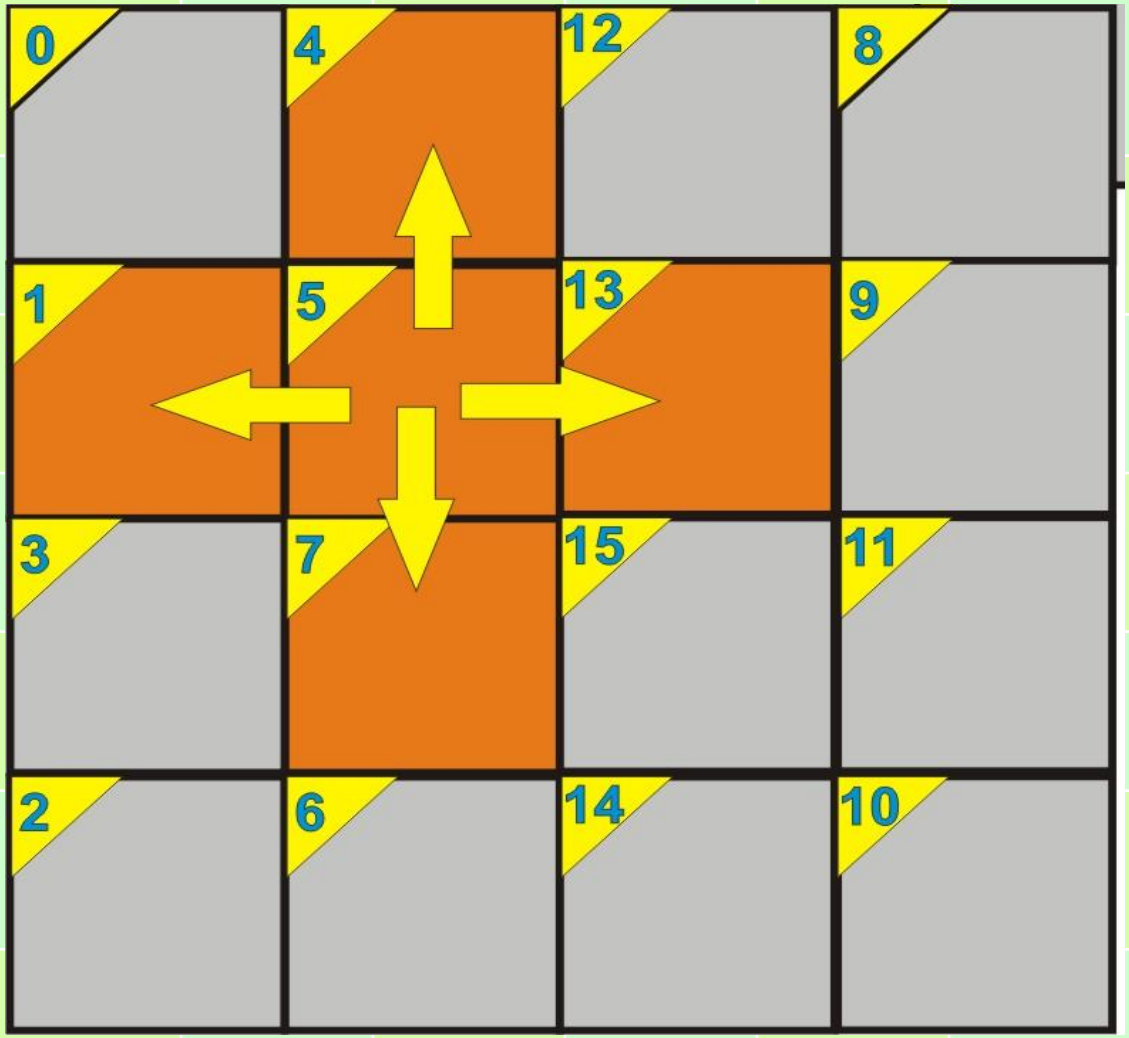
Kmap para 5 variables

Con 5 Variables se tienen ___ términos

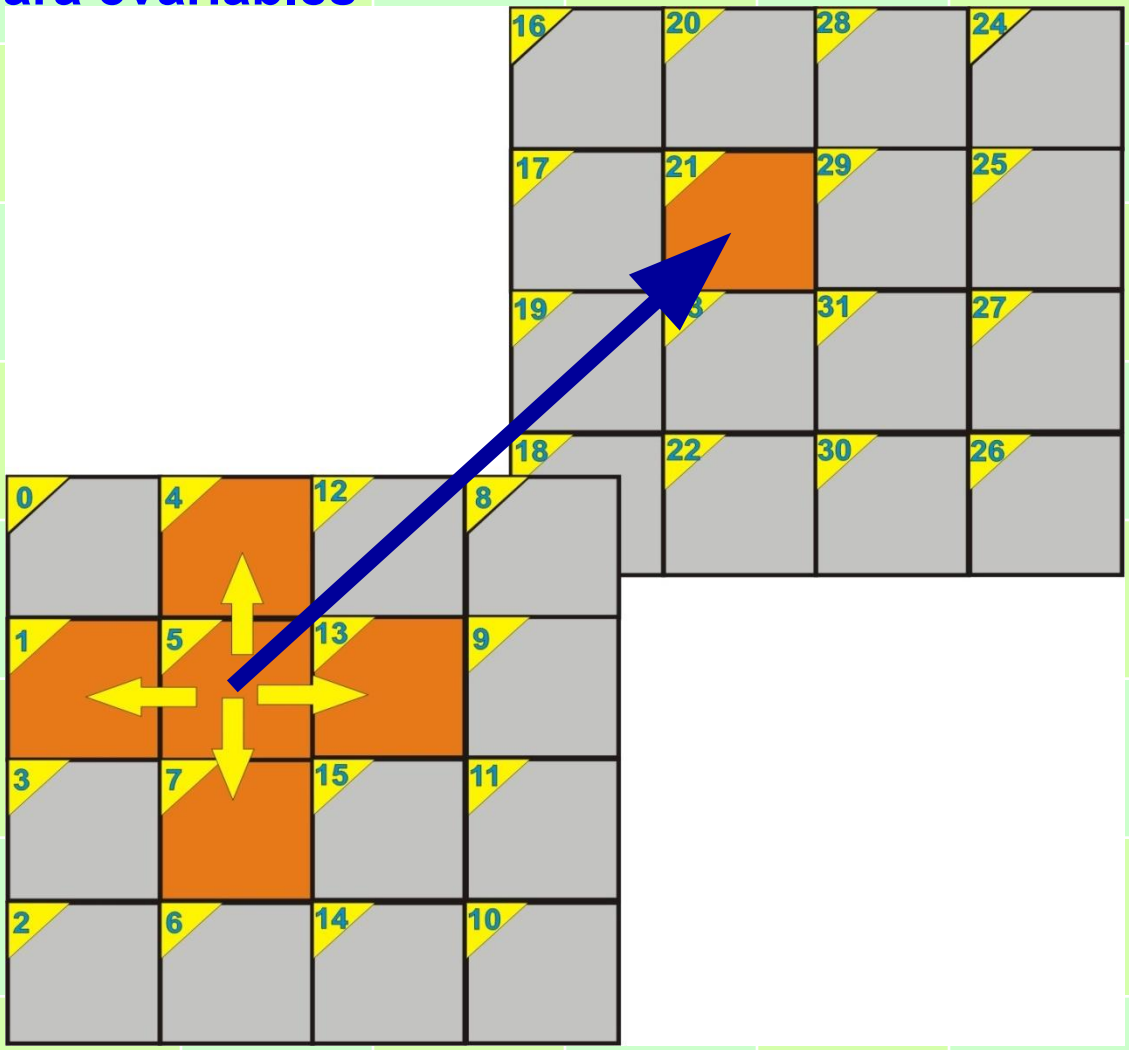
y cada termino tiene 5 posibilidades ___ de factorización



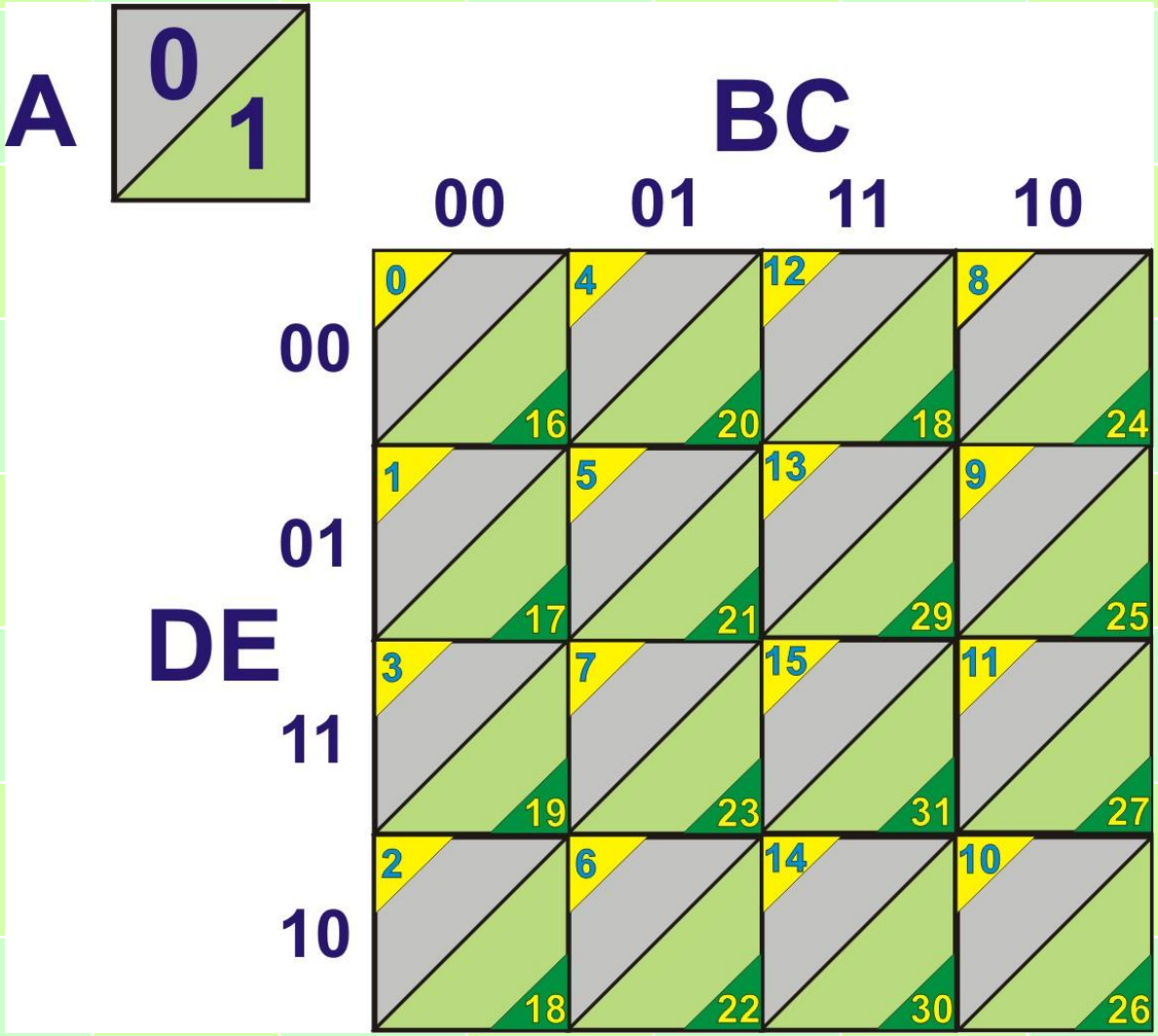
Kmap para 5variables



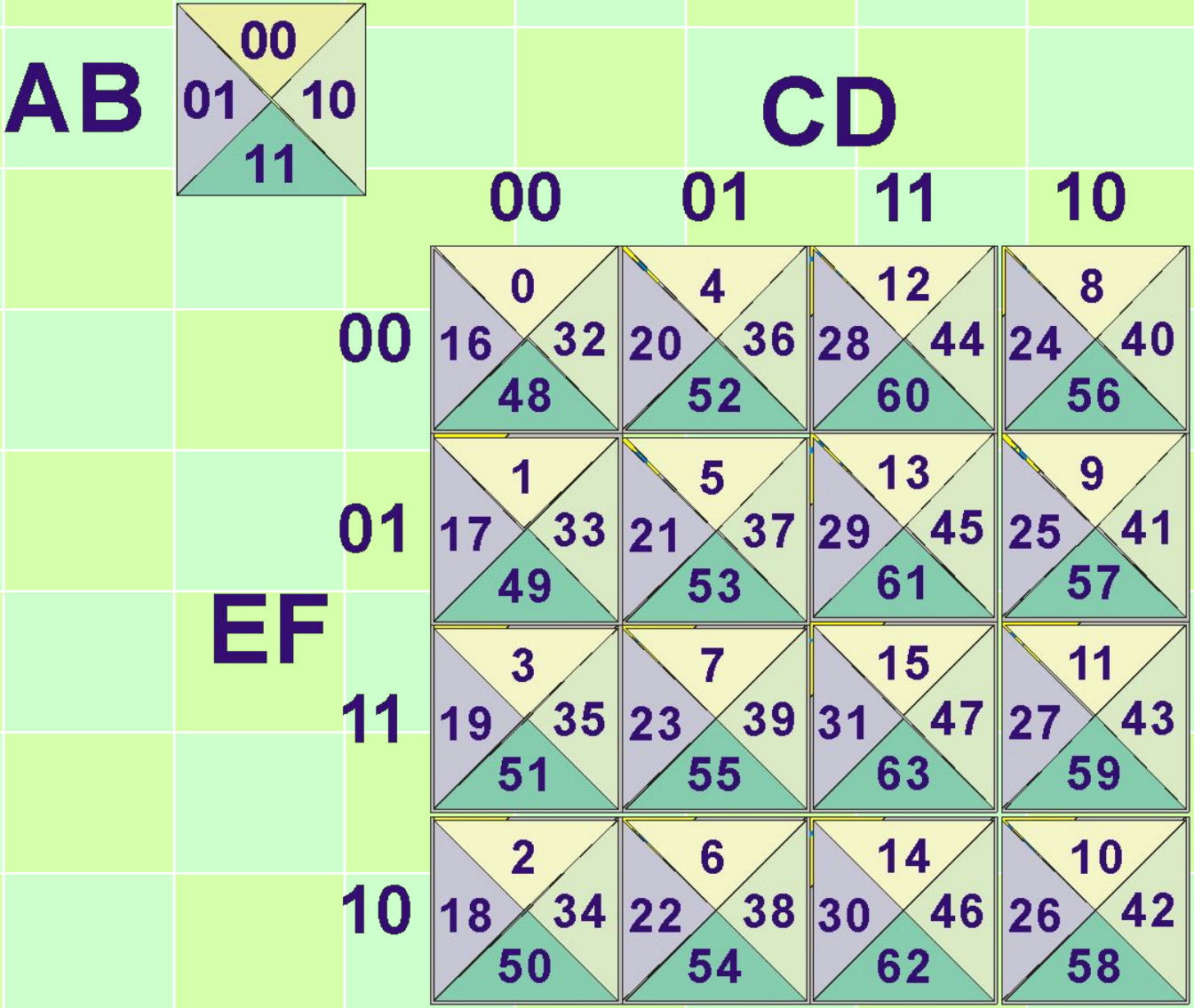
Kmap para 5variables



Kmap para 5 variables



Kmap para 6 variables



Reglas para el uso del Kmap

1.- Formar el menor numero de grupos

2.- Cada grupo lo mas grande posible

3.- Todos los unos deberán de ser agrupados

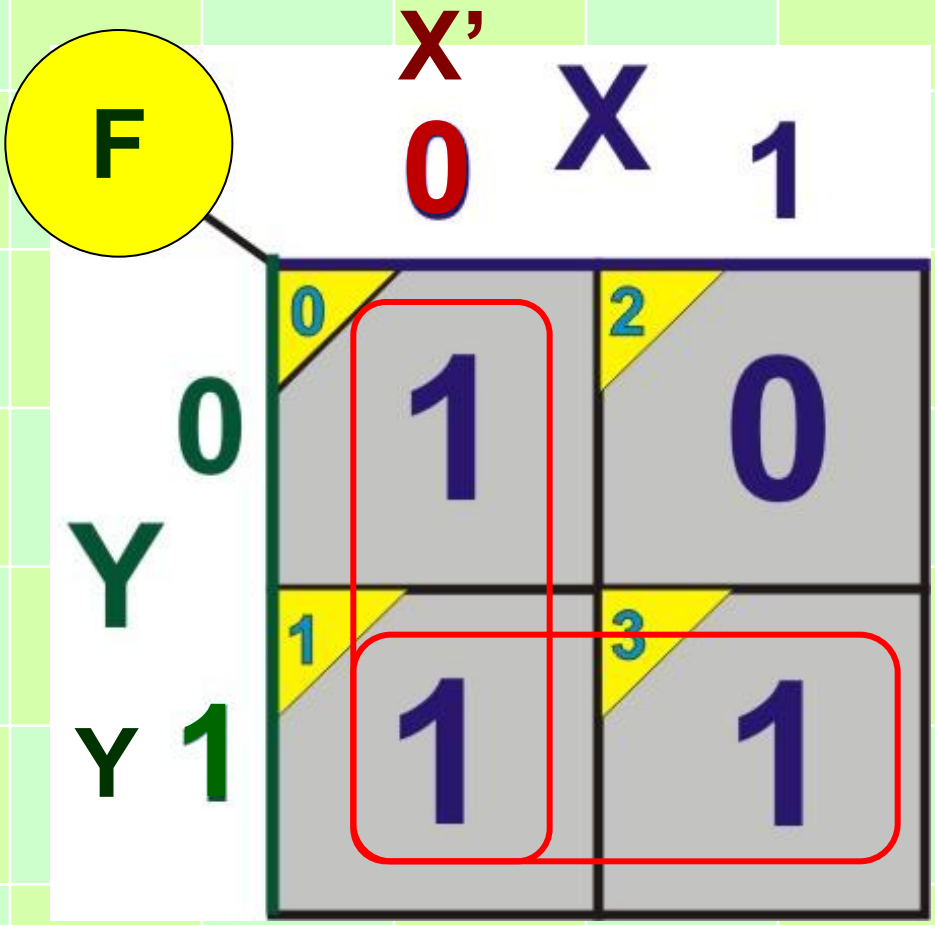
4.- Un solo uno puede formar un grupo

5.- Casillas de un grupo pueden formar parte de otro grupo

Grupo = Unos adyacentes enlazados (paralelogramos) en una cantidad igual a una potencia entera de dos (ejemplo 1, 2, 4, 8,…).

Ejemplos del Kmap

m	X	Y	F
0	0	0	1
1	0	1	1
2	1	0	0
3	1	1	1



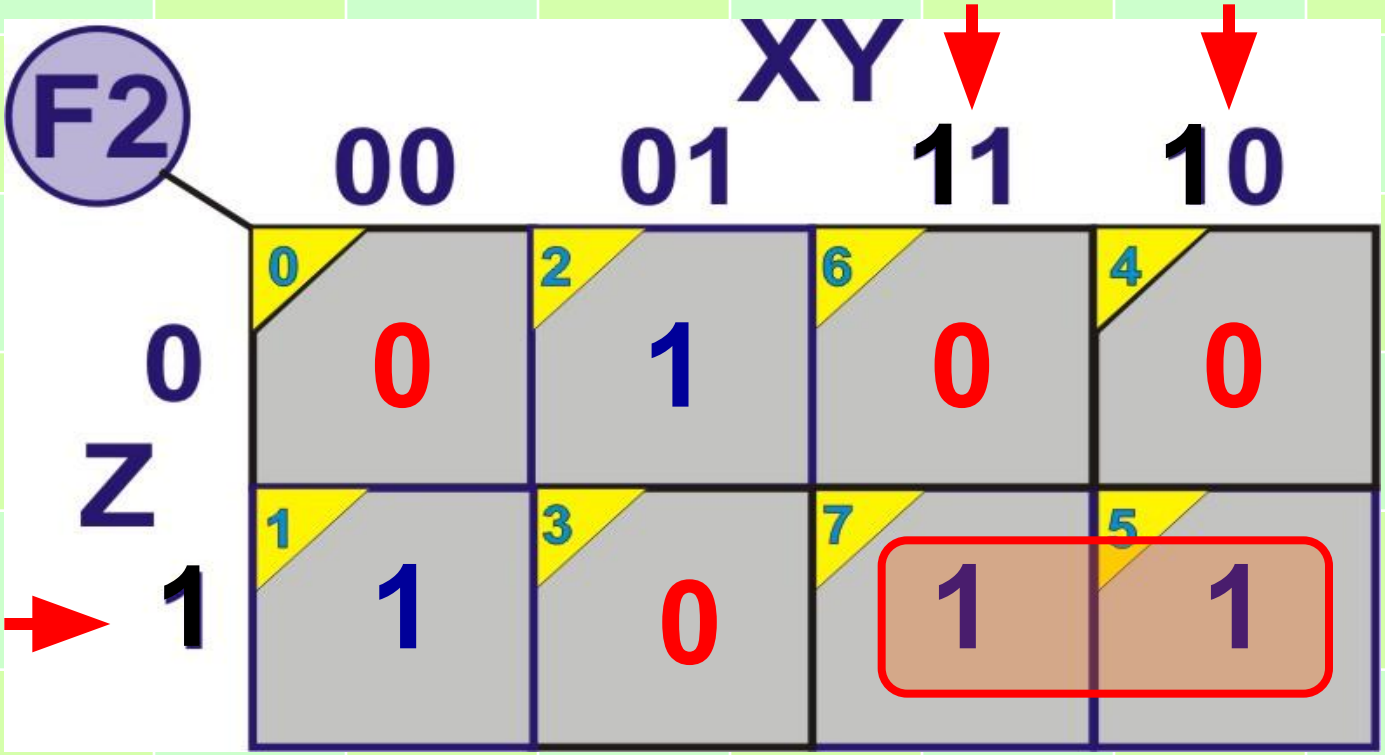
$$F_{(X,\,Y)} = X' + Y$$

$$F2_{(X, Y, Z)} = \Sigma m(1, 2, 5, 7)$$

F2	XY 00	01	11	10
Z 0	0 (0)	1 (2)	0 (6)	0 (4)
Z 1	1 (1)	0 (3)	1 (7)	1 (5)

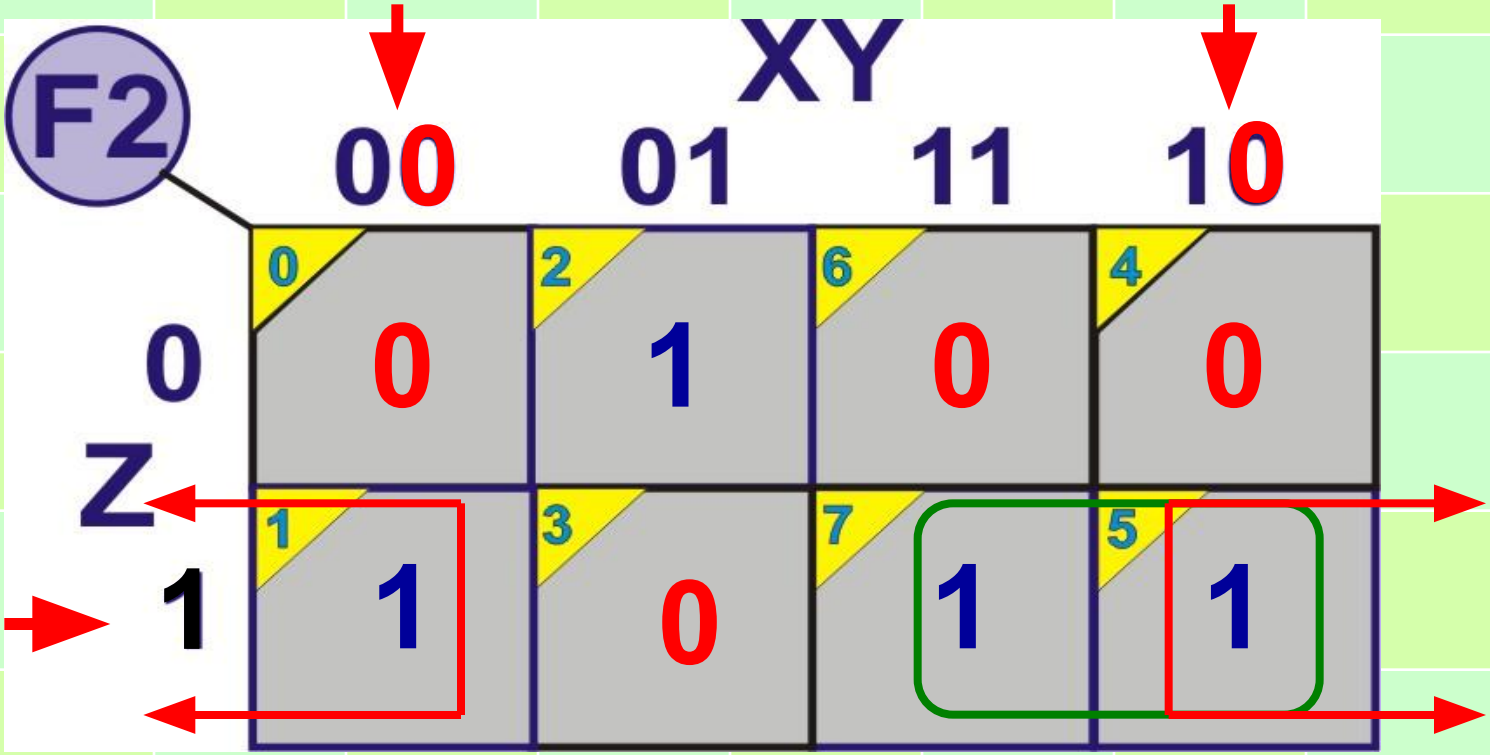
ejemplos del Kmap

$$F2_{(X, Y, Z)} = \Sigma m(1, 2, 5, 7)$$



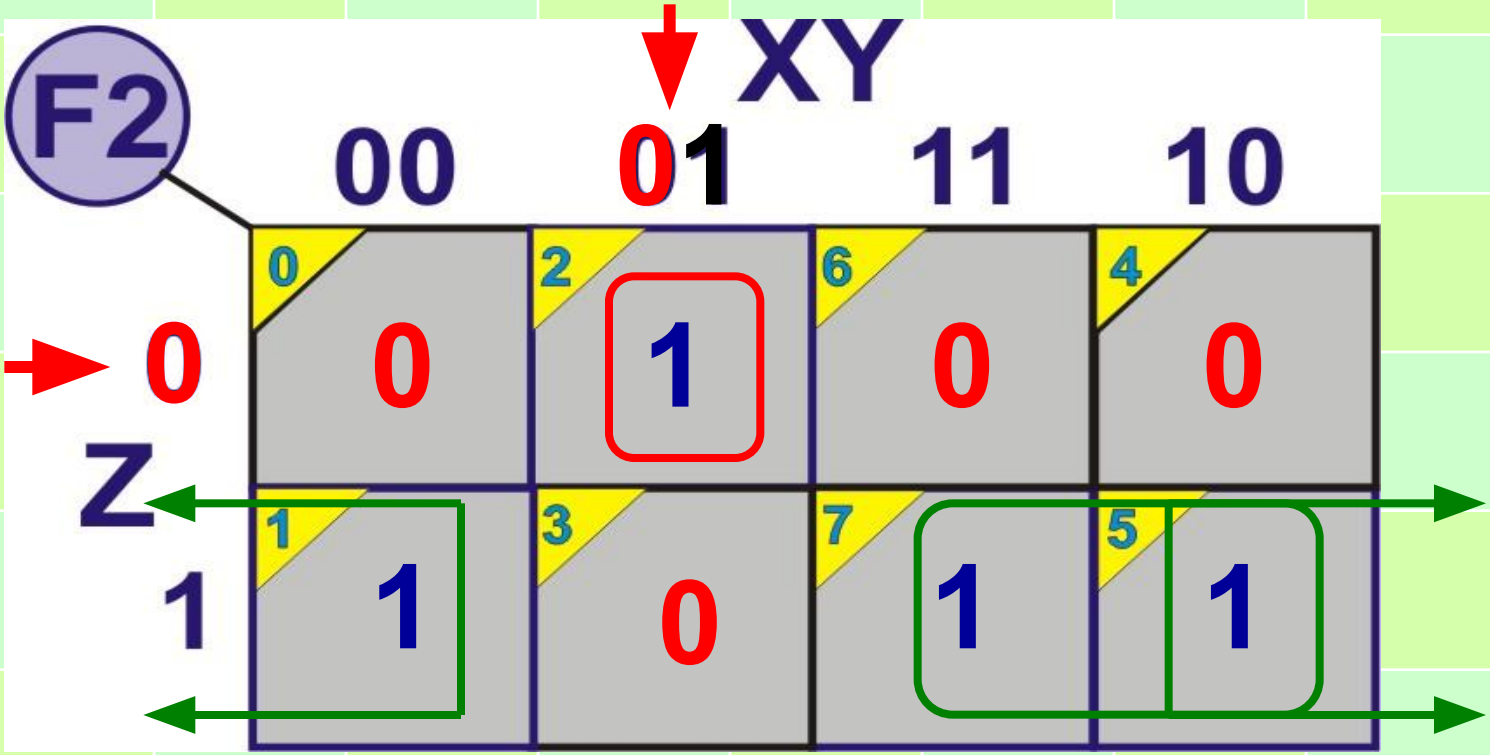
$$F2_{(X, Y, Z)} = X\ Z +$$

$$F2_{(X, Y, Z)} = \Sigma m(1, 2, 5, 7)$$



$$F2_{(X, Y, Z)} = X\ Z + Y'\ Z$$

$$F2_{(X, Y, Z)} = \Sigma m(1, 2, 5, 7)$$



$$F2_{(X, Y, Z)} = X\ Z + Y'\ Z + X'\ Y\ Z'$$

And/Or

$$F2_{(X, Y, Z)} = \Sigma m(1, 2, 5, 7)$$

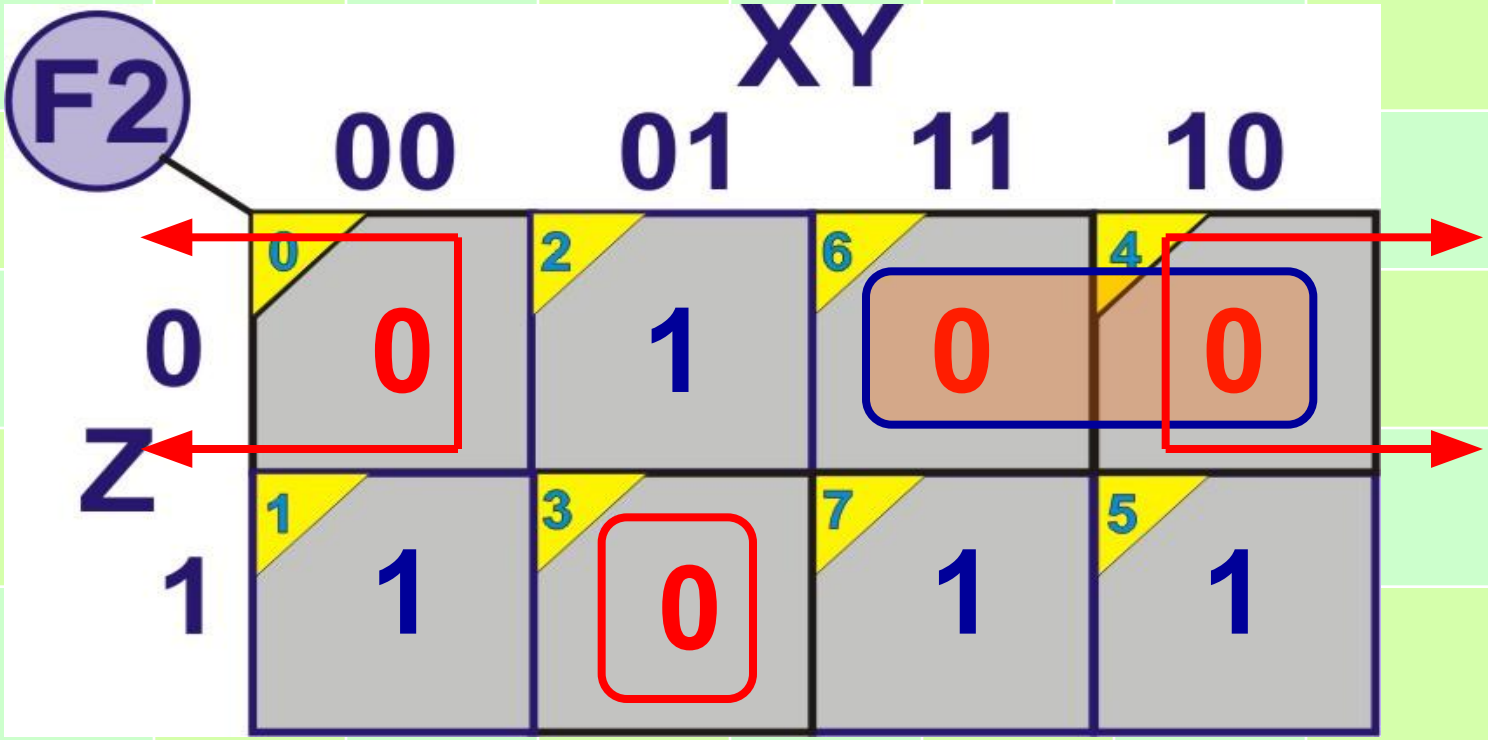
Agrupando ceros

F2	XY: 00	01	11	10
Z: 0	0 (0)	1 (2)	0 (6)	0 (4)
Z: 1	1 (1)	0 (3)	1 (7)	1 (5)

$$F2_{(X, Y, Z)} = X\ Z' +$$

$$F2_{(X, Y, Z)} = \Sigma m(1, 2, 5, 7)$$

Agrupando ceros



$$F2_{(X, Y, Z)} = \overline{X\ Z' + Y'\ Z' + X'\ Y\ Z}$$

And/Nor

$$F2_{(X, Y, Z)} = \Sigma m(1, 2, 5, 7)$$

Resultados del Kmap

Agrupando unos
And/Or

$$F2_{(X, Y, Z)} = X\,Z + Y'\,Z + X'\,Y\,Z'$$

Agrupando ceros
And/Nor

$$F2_{(X, Y, Z)} = \overline{X\,Z' + Y'\,Z' + X\,Y'\,Z'}$$

Aplicando el teorema de D'Morgan
Or/And

$$F2_{(X, Y, Z)} = (X'+Z)(Y+Z)(X'+Y+Z)$$

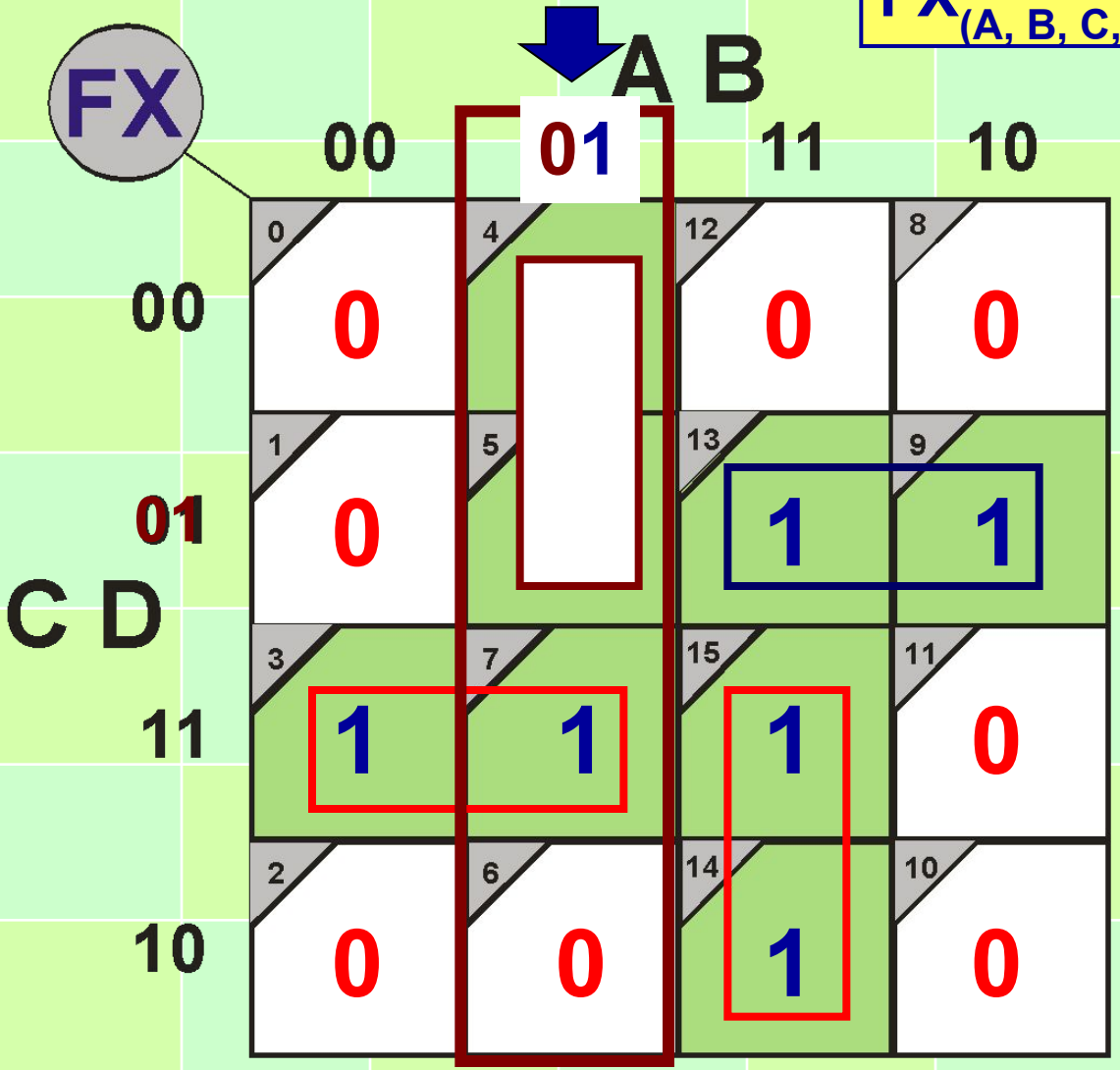
$$FX_{(A,\ B,\ C,\ D)} = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$$

FX

C D \ A B	00	01	11	10
00	0 (0)	1 (4)	0 (12)	0 (8)
01	0 (1)	1 (5)	1 (13)	1 (9)
11	1 (3)	1 (7)	1 (15)	0 (11)
10	0 (2)	0 (6)	1 (14)	0 (10)

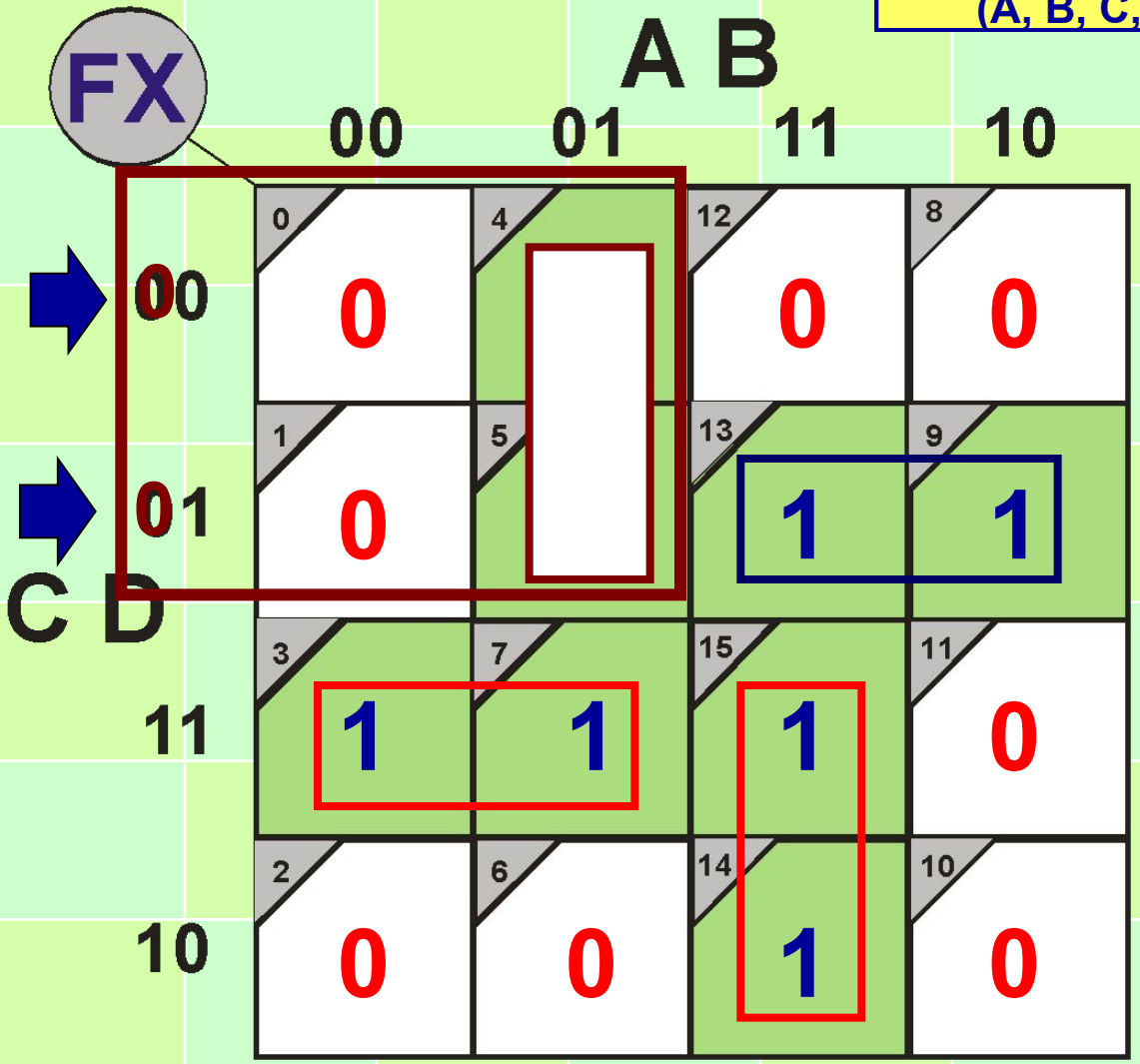
$FX_{(A, B, C, D)} = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$

$FX_{(A, B, C, D)} = A' B$



$FX_{(A, B, C, D)} = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$

$FX_{(A, B, C, D)} = A' B C'$



$FX_{(A, B, C, D)} = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$

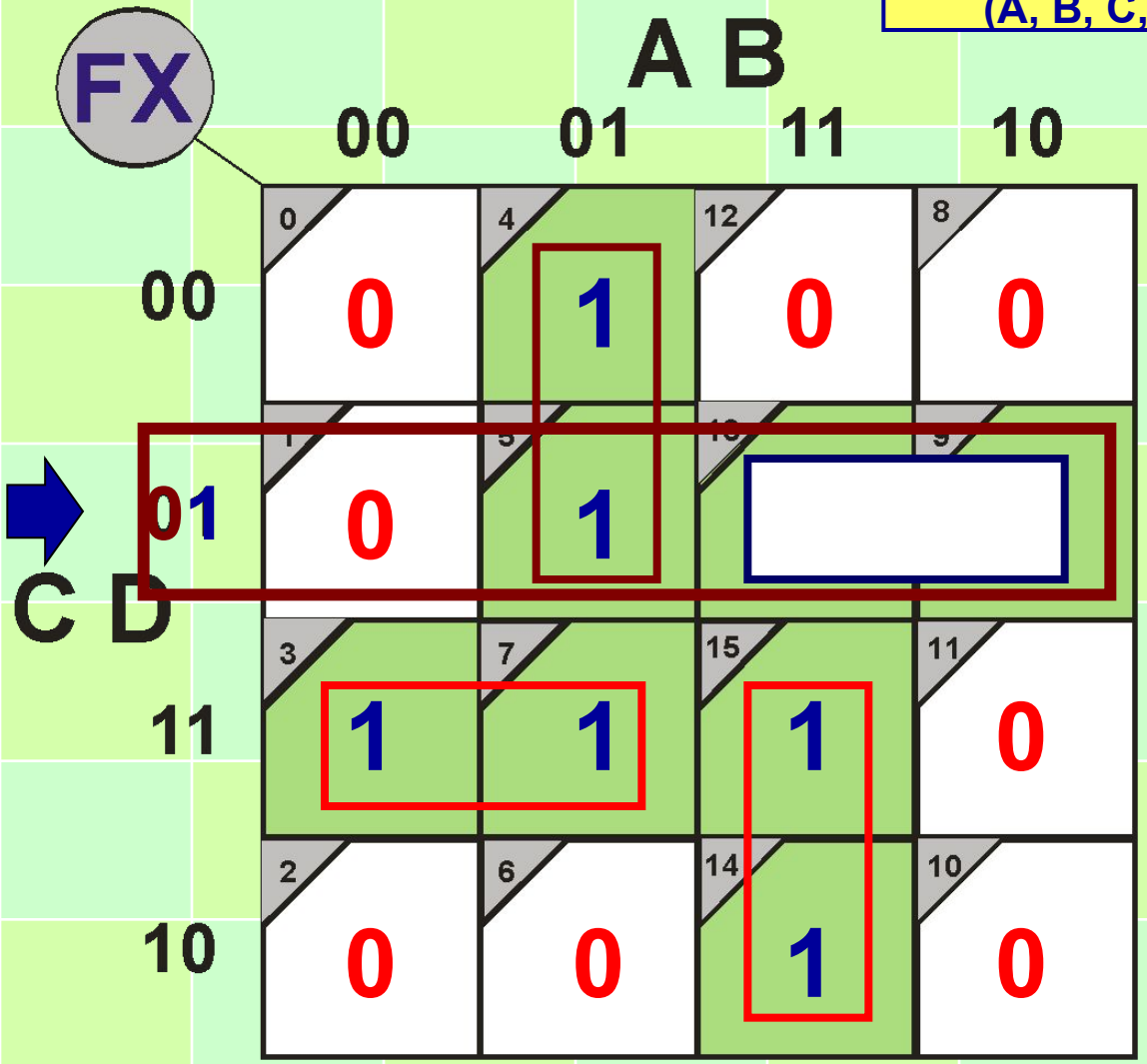
$FX_{(A, B, C, D)} = A' B C' + A$

FX

C D \ A B	00	01	11	10
00	0	1	0	0
01	0	1		
11	1	1	1	0
10	0	0	1	0

$FX_{(A, B, C, D)} = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$

$FX_{(A, B, C, D)} = A' B C' + A C'D$



$FX_{(A, B, C, D)} = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$

$FX_{(A, B, C, D)} = A' B C' + A C'D + A'$

FX

C D \ A B	00	01	11	10
00	0 (0)	1 (4)	0 (12)	0 (8)
01	0 (1)	1 (5)	1 (13)	1 (9)
11	(3)	(7)	1 (15)	0 (11)
10	0 (2)	0 (6)	1 (14)	0 (10)

$FX_{(A, B, C, D)} = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$

$FX_{(A, B, C, D)} = A' B C' + A C'D + A' CD$

FX

C D \ A B	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	1	1	1	0
10	0	0	1	0

$FX_{(A, B, C, D)} = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$

$FX_{(A, B, C, D)} = A' B C' + A C'D + A'CD + AB$

FX

C D \ A B	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	1	1		0
10	0	0		0

$FX_{(A, B, C, D)} = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$

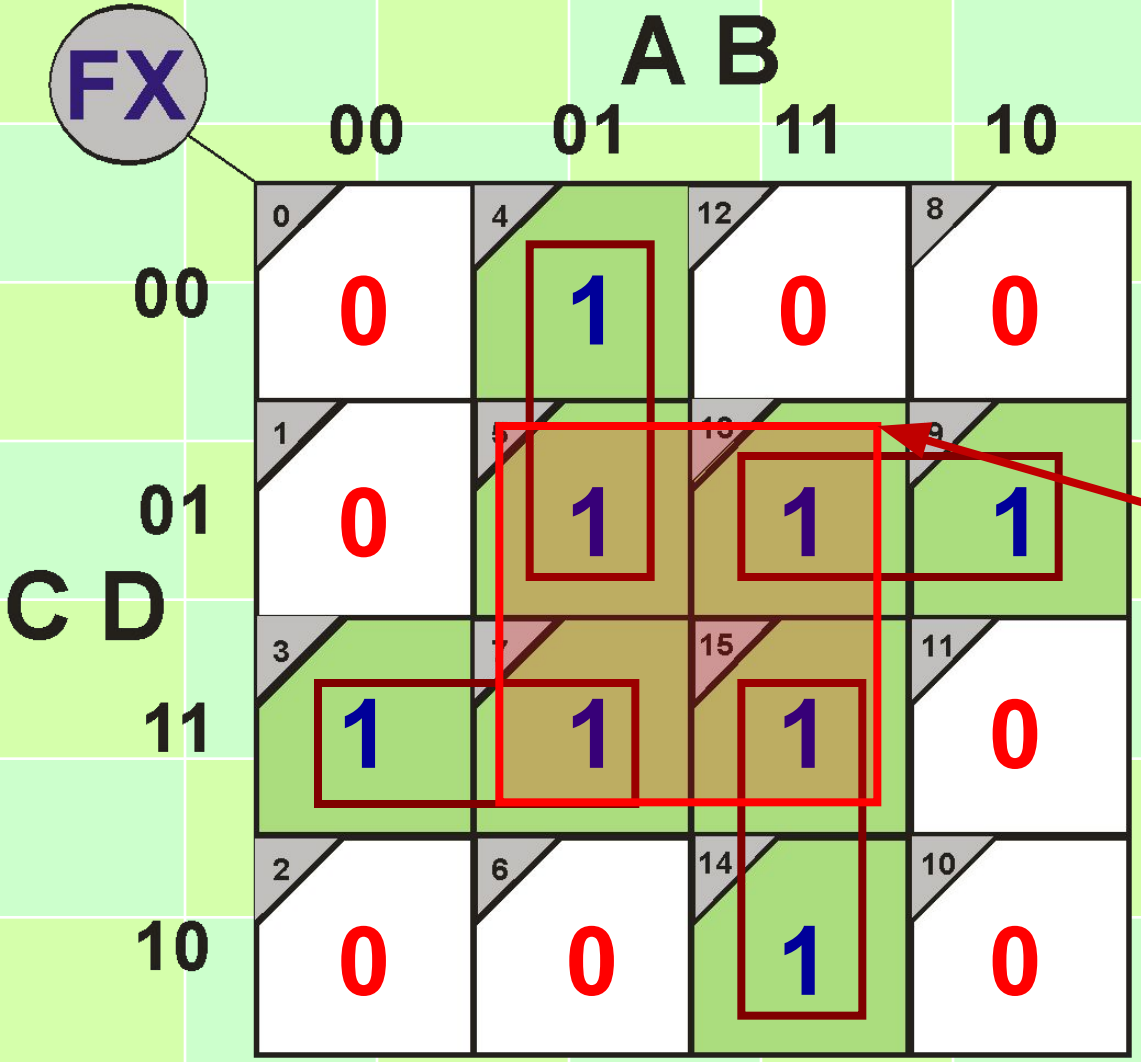
$FX_{(A, B, C, D)} = A' B C' + A C'D + A'CD + AB C$

FX

C D \ A B	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	1	1		0
10	0	0		0

$$FX_{(A, B, C, D)} = A' B C' + A C'D + A'CD + ABC$$

And/Or

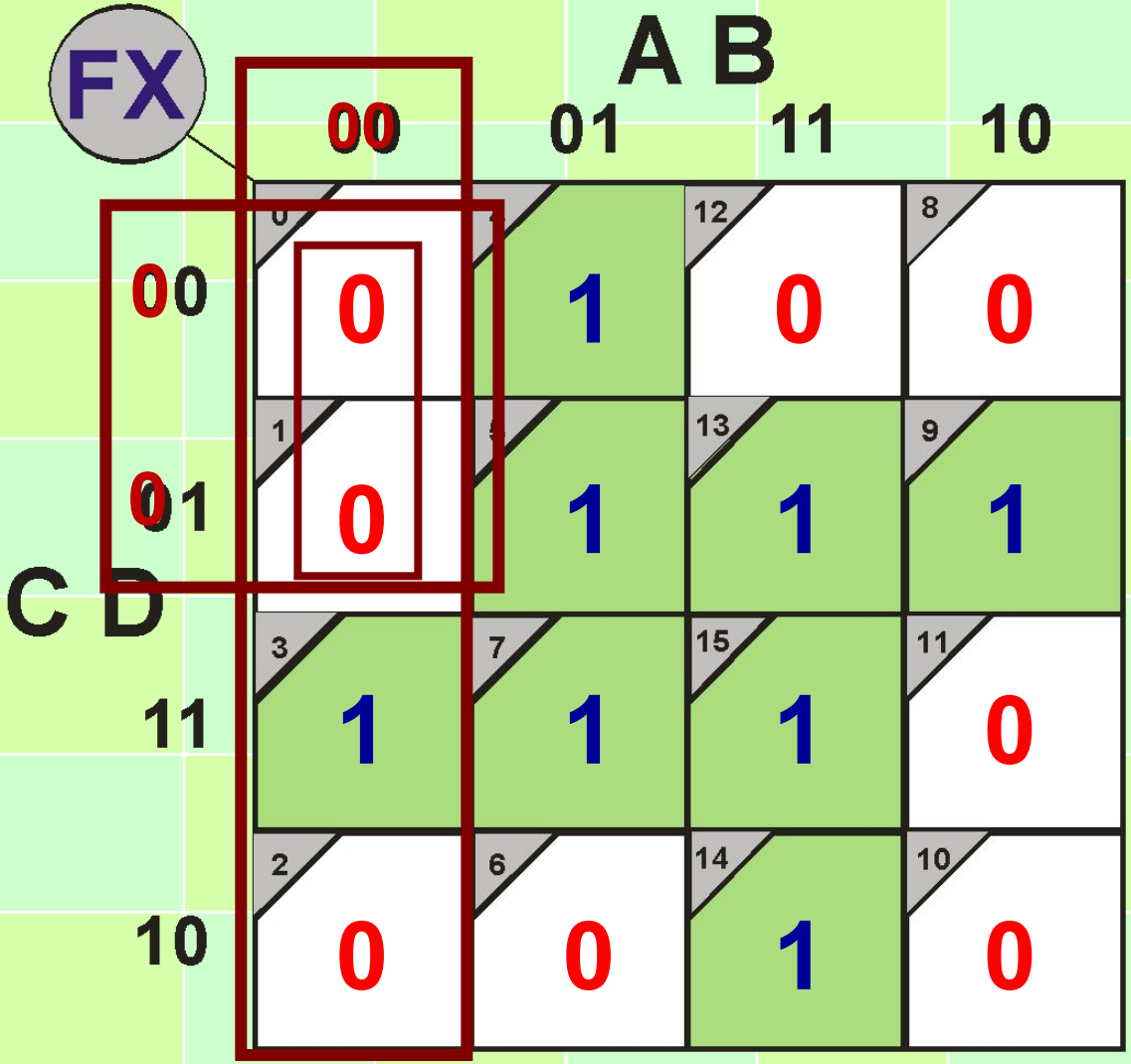


1.- Formar el menor número de grupos

2.- Cada grupo lo más grande posible

Lazo redundante

$$FX_{(A, B, C, D)} = A' B' C'$$



Agrupando ceros

And/Nor

$$FX_{(A, B, C, D)} = A' B' C' + A' C D' + A C' D' + A B' C$$

And/Nor

FX

C D \ A B	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	1	1	1	0
10	0	0	1	0

$$FX_{(A, B, C, D)} = A' B' C' + A' C D' + A C' D' + A B' C$$

$$(A+B+C)\ (A+C'+D)(A'+C+D)(A'+B+C')$$

Or/And

$$FY_{(A, B, C, D)} = \Pi\ m\ (1, 3, 8, 9, 11, 12, 14)$$

FY

C D \ A B	00	01	11	10
00	0	4	12: 0	8: 0
01	1: 0	5	13	9: 0
11	3: 0	7	15	11: 0
10	2	6	14: 0	10

$$FY_{(A, B, C, D)} = \Pi\, m\,(1, 3, 8, 9, 11, 12, 14)$$

FY

C D \ A B	00	01	11	10
00	0: 1	4: 1	12: 0	8: 0
01	1: 0	5: 1	13: 1	9: 0
11	3: 0	7: 1	15: 1	11: 0
10	2: 1	6: 1	14: 0	10: 1

1.- Formar el menor número de grupos

2.- Cada grupo lo más grande posible

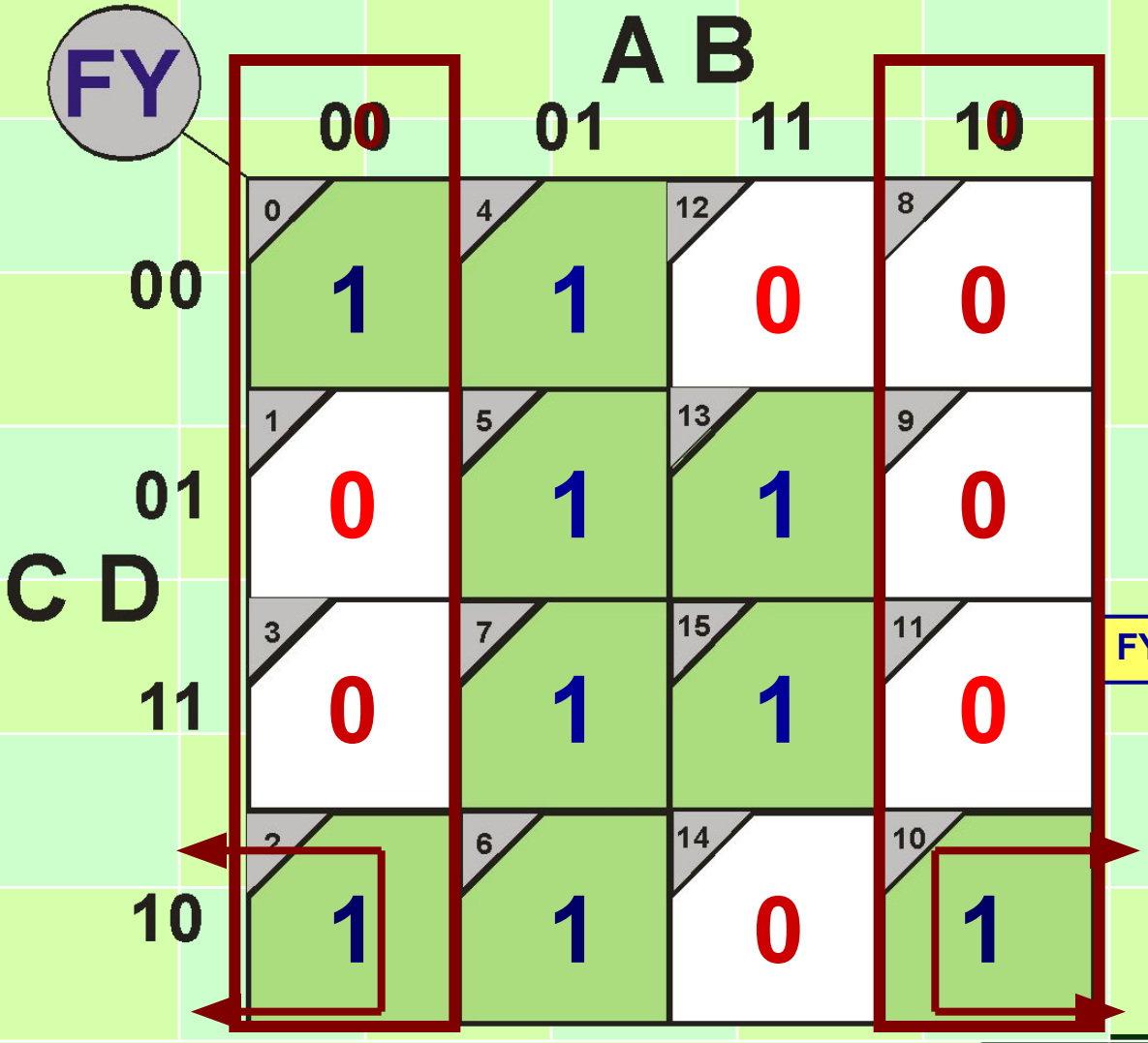
$FY_{(A, B, C, D)} =$

FY

C D \ A B	00	01	11	10
00	0: 1	4: 1	12: 0	8: 0
01	1: 0	5: 1	13: 1	9: 0
11	3: 0	7: 1	15: 1	11: 0
10	2: 1	6: 1	14: 0	10: 1

$FY_{(A, B, C, D)} = \Pi\ m\ (1, 3, 8, 9, 11, 12, 14)$

$$FY_{(A, B, C, D)} = B'$$



$$FY_{(A, B, C, D)} = \Pi\ m\ (1, 3, 8, 9, 11, 12, 14)$$

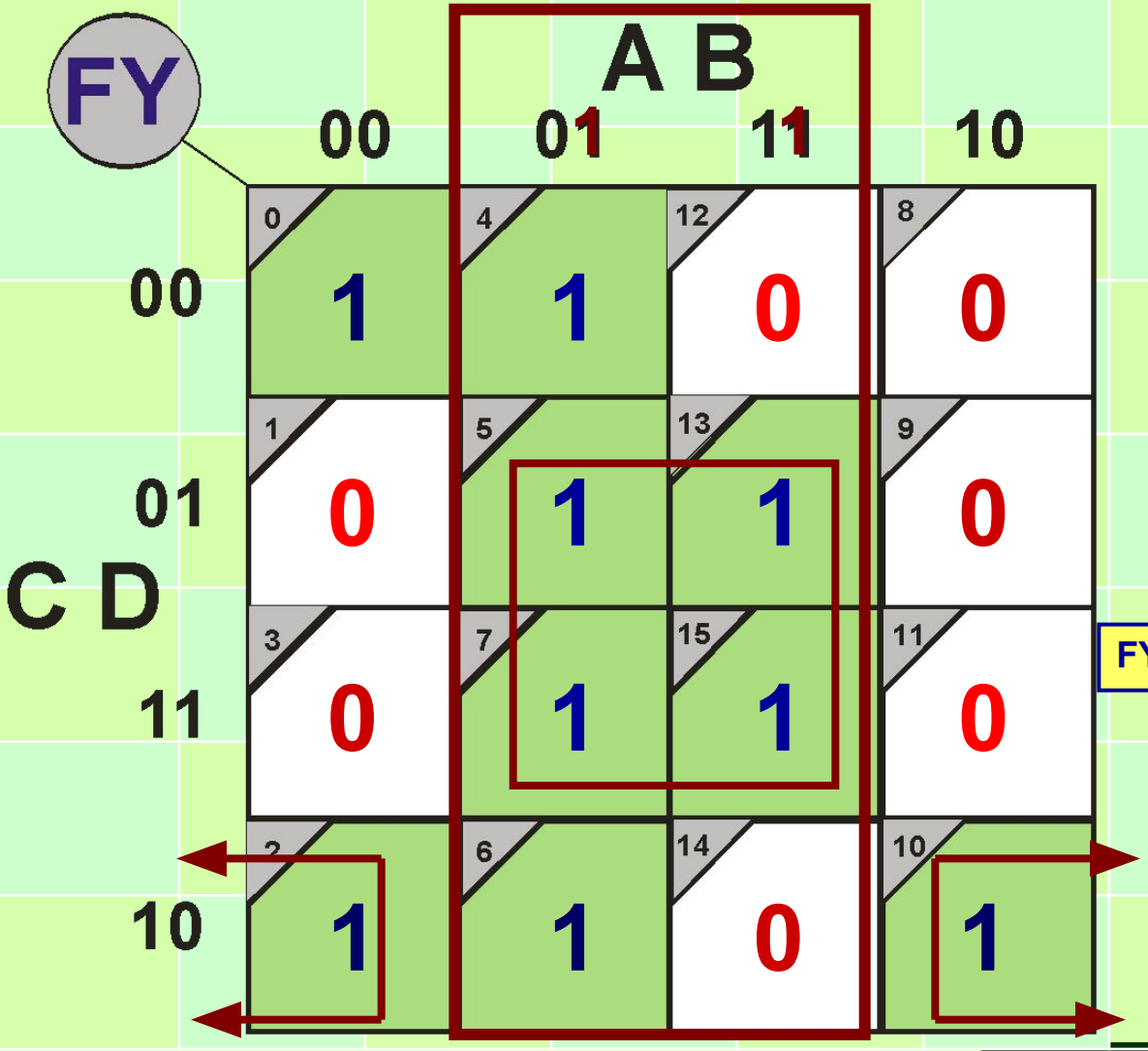
$$FY_{(A, B, C, D)} = B' CD'$$

FY

C D \ A B	00	01	11	10
00	0: 1	4: 1	12: 0	8: 0
01	1: 0	5: 1	13: 1	9: 0
11	3: 0	7: 1	15: 1	11: 0
10	2: 1	6: 1	14: 0	10: 1

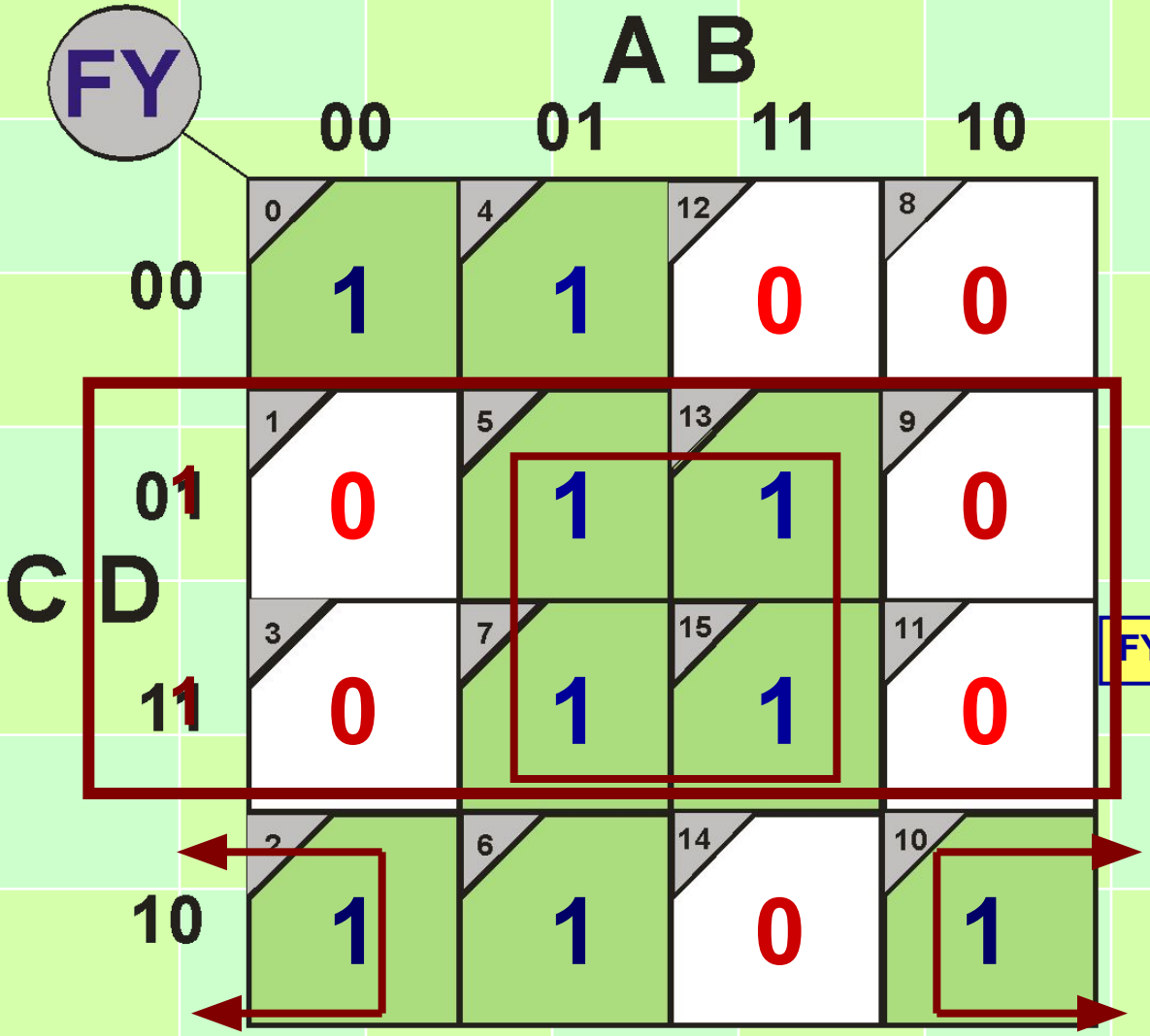
$$FY_{(A, B, C, D)} = \Pi\ m\ (1, 3, 8, 9, 11, 12, 14)$$

$$FY_{(A, B, C, D)} = B' C D' + B$$



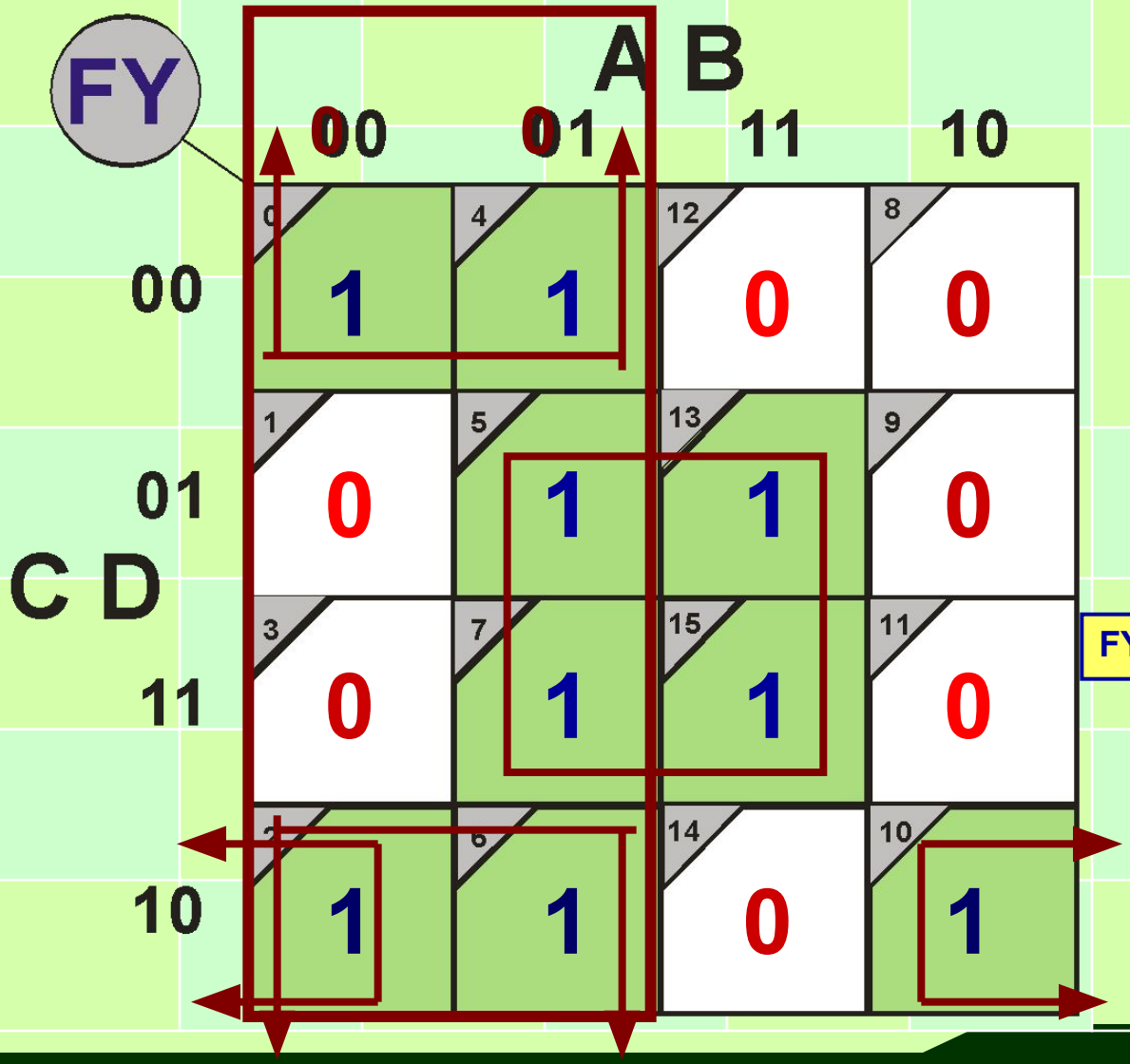
$$FY_{(A, B, C, D)} = \Pi\ m\ (1, 3, 8, 9, 11, 12, 14)$$

$$FY_{(A, B, C, D)} = B' C D' + B D$$



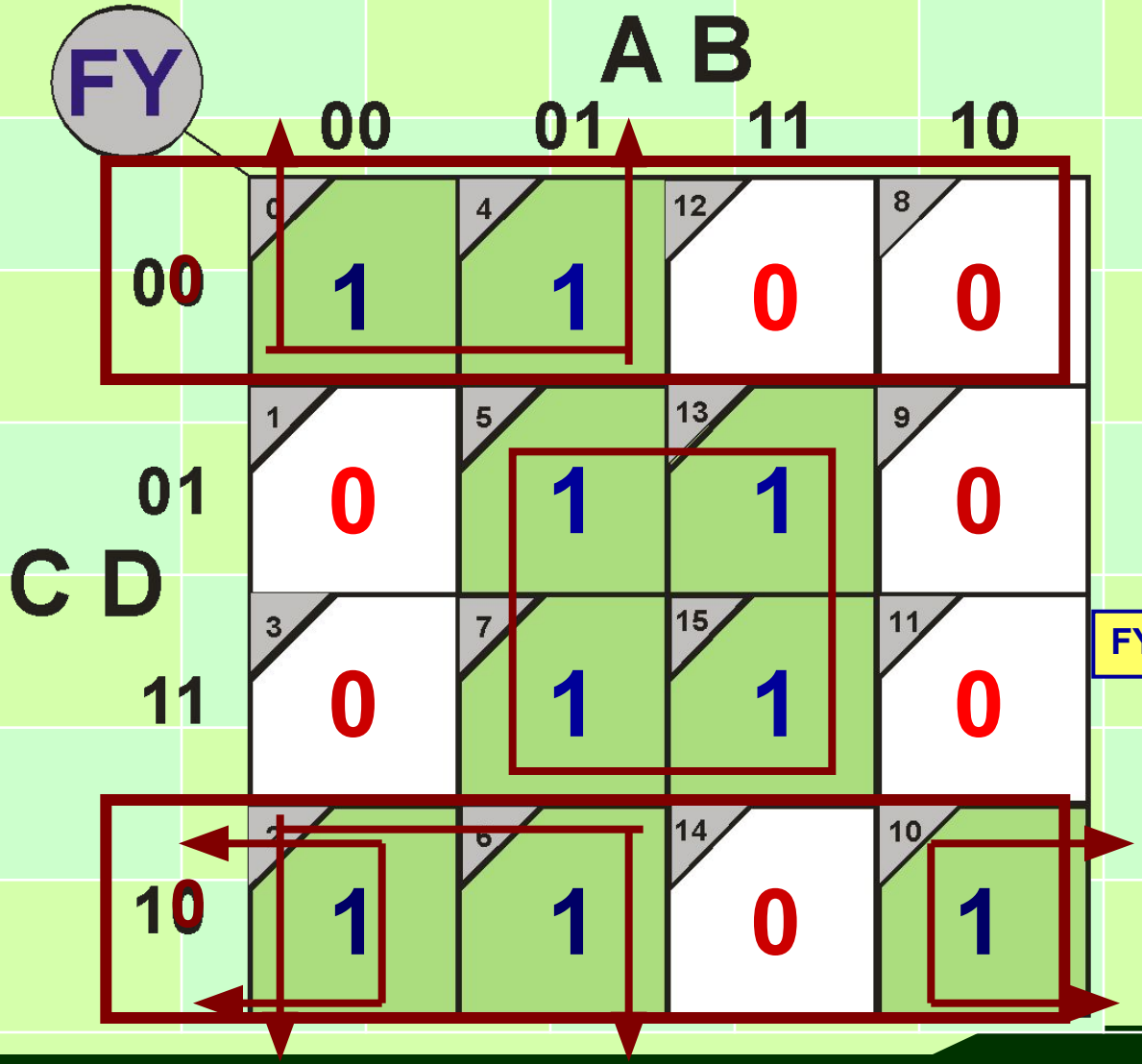
$$FY_{(A, B, C, D)} = \Pi\ m\ (1, 3, 8, 9, 11, 12, 14)$$

$$FY_{(A, B, C, D)} = B' C D' + B D + A'$$



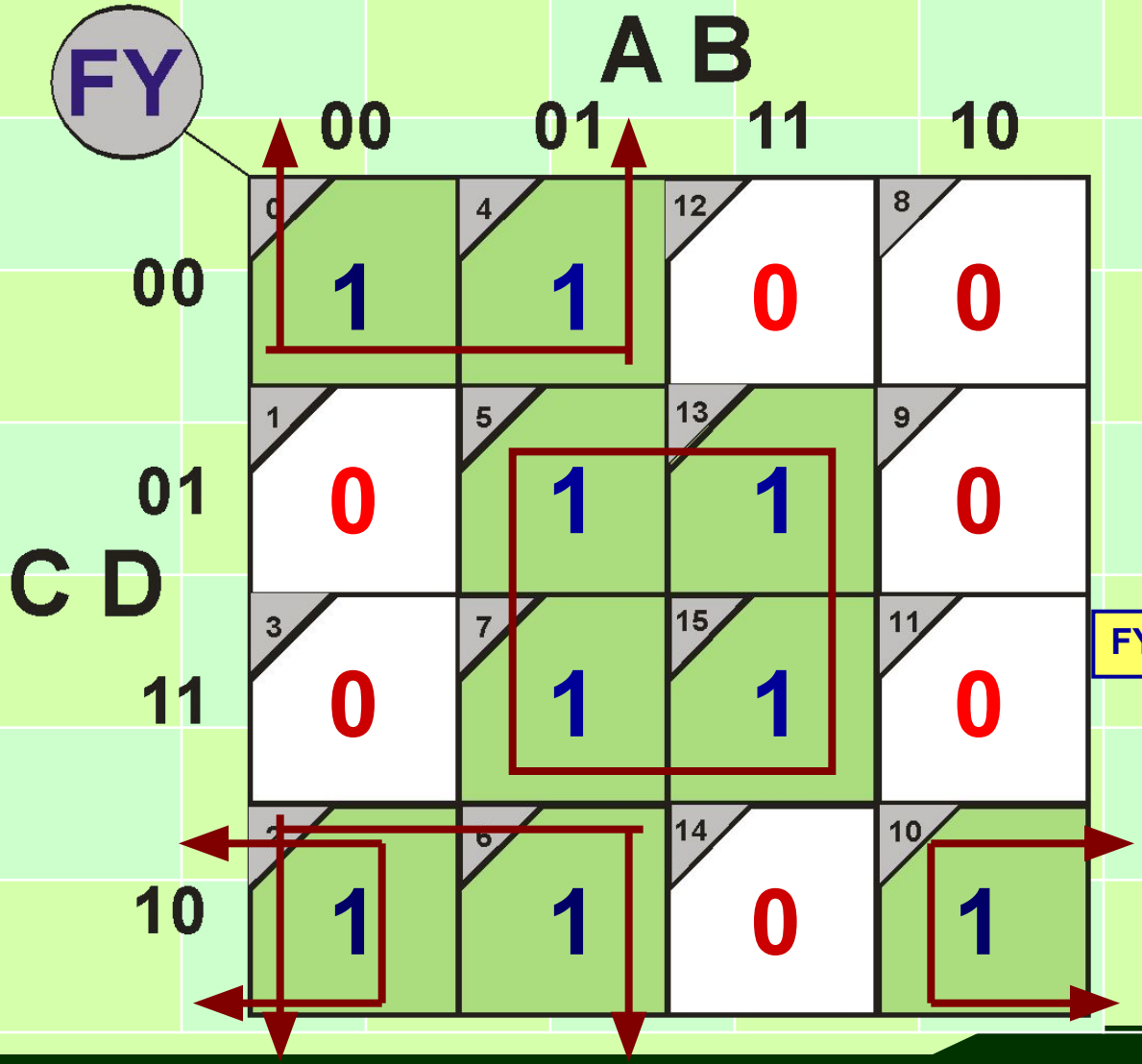
$$FY_{(A, B, C, D)} = \Pi\ m\ (1, 3, 8, 9, 11, 12, 14)$$

$$FY_{(A, B, C, D)} = B' C D' + B D + A' D'$$



$$FY_{(A, B, C, D)} = \Pi\ m\ (1, 3, 8, 9, 11, 12, 14)$$

$$FY_{(A, B, C, D)} = B' C D' + B D + A' D'$$



$$FY_{(A, B, C, D)} = \Pi\ m\ (1, 3, 8, 9, 11, 12, 14)$$

$$F_{(X,\,Y,\,Z,\,W)} = \Sigma m(2,3,5,6,7,10,11,14,15)$$

F8

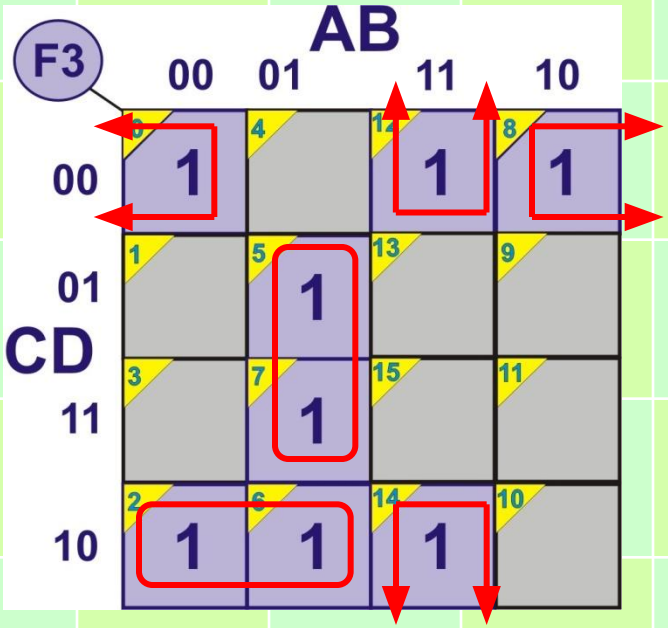
ZW \ XY	00	01	11	10
00	0 (0)	0 (4)	0 (12)	0 (8)
01	0 (1)	1 (5)	0 (13)	0 (9)
11	1 (3)	1 (7)	1 (15)	1 (11)
10	1 (2)	1 (6)	1 (14)	1 (10)

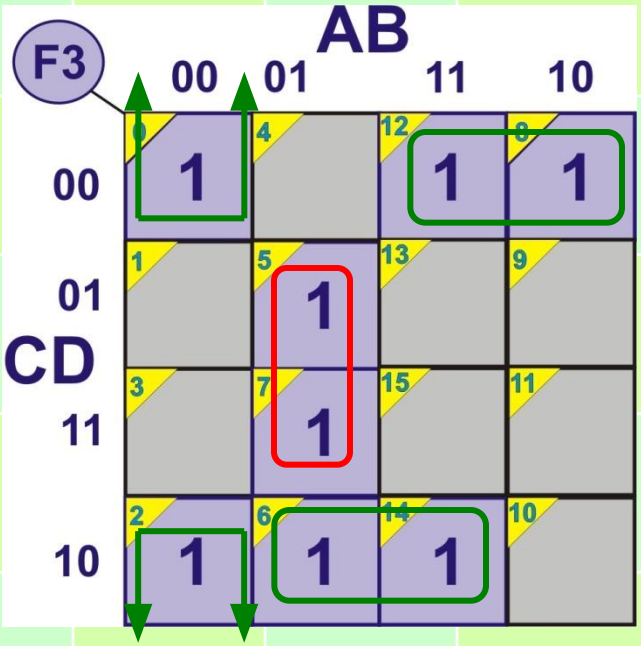
$$F3_{(A,\ B,\ C,\ D)} = \Sigma m(0,2,5,6,7,8,12,14)$$

F3	AB: 00	01	11	10
CD: 00	0: 1	4	12: 1	8: 1
01	1	5: 1	13	9
11	3	7: 1	15	11
10	2: 1	6: 1	14: 1	10

$F3_{(A, B, C, D)} = \Sigma m(0,2,5,6,7,8,12,14)$

$$F3 = A'B'D' + A\,C'D' + A'B\,D + B\,C\,D'$$
$$F3 = B'C'D' + A'C\,D' + A'B\,D + A\,B\,D'$$



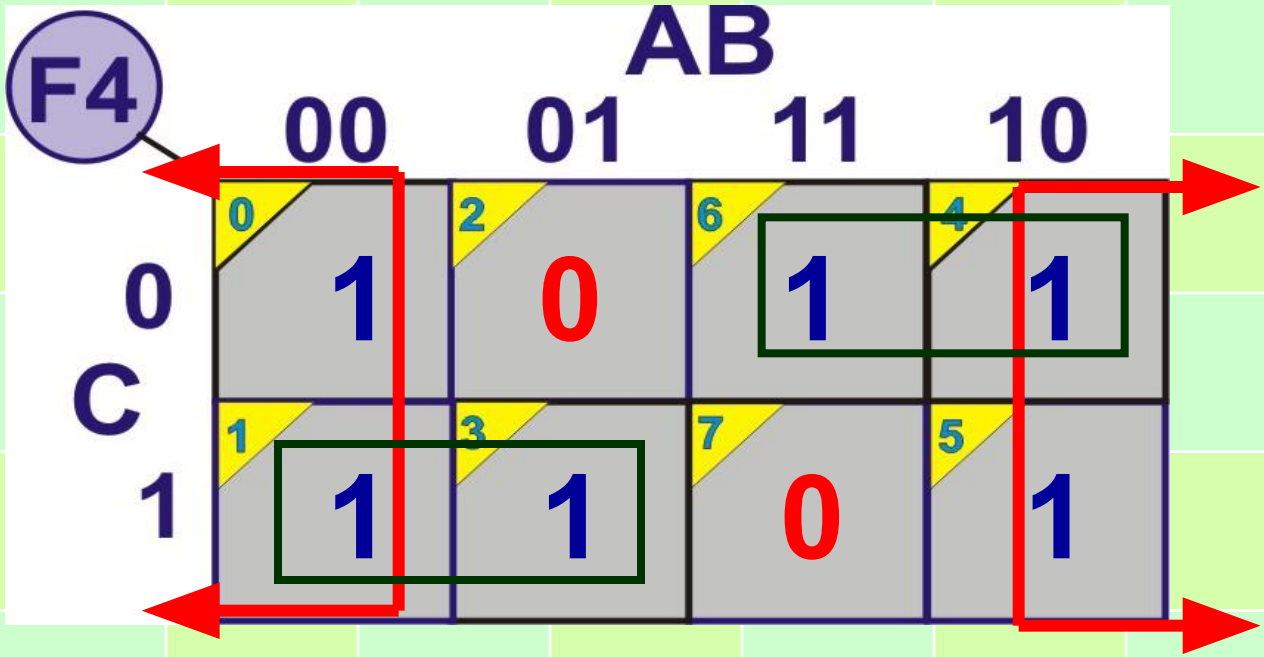


$$F4_{(A,\ B,\ C)} = \Pi m(2,\ 7)$$

F4

C \ AB	00	01	11	10
0	0: 1	2: 0	6: 1	4: 1
1	1: 1	3: 1	7: 0	5: 1

$$F4_{(A,\ B,\ C)} = \Pi m(2,\ 7)$$



$$F4(A,\ B,\ C) = B' + A\ C' + A'C$$

Temas de examen y ponderación

Conceptos Digital y Analógico Sistemas Numéricos	10%
Álgebra Booleana Operadores (And, Or, Not, Nand, Nor, Exor y Exnor) Símbolo, característica, expresión matemática, Tabla de verdad Propiedades Distributiva, Asociativa y Conmutativa e Identidades Circuito, Ecuación, Tabla de verdad y Diagrama de tiempos Teorema de D' Morgan Minitermino forma SOP Maxitermino forma POS	50%
Minimización de Funciones Booleanas Manipulación Algebraica Mapas de Karnaugh POS y SOP	40%

Instrucciones para el examen de medio curso

a) Lea cuidadosamente el contenido y lo solicitado en el examen.

b) Utilice hojas adicionales para las respuestas, indicando para cada problema el procedimiento efectuado, así como indicar claramente la solución obtenida **encerrada en un rectángulo**.

c

Instrucciones para el examen de medio curso

c) **Confirme los resultados obtenidos, un procedimiento sin resultados correctos carece de valor.**

d) **Un resultado correcto sin documentar adecuadamente el procedimiento realizado carece de credibilidad.**

e) Concéntrese en su examen, tiene 2.15 horas para resolverlo.

Instrucciones para el examen de medio curso

Nota: se anexa formulario en este documento.

f) Al terminar su examen digitalice las hojas de respuestas y anéxelas al archivo PDF de este examen, y envíalas en formato PDF junto con los archivos entregables solicitados a la plataforma Google Classroom, en la sección correspondiente a examen de medio curso, en un solo archivo comprimido con el nombre de la hora y numero de lista ejemplo M1NL25.zip.

Instrucciones para el examen de medio curso

g) Las respuestas enviadas después de haber concluido el tiempo programado para este examen no serán evaluadas.

h) En caso necesario el profesor te agendará una entrevista, para efectuar la revisión de los procedimientos efectuados y resultados obtenidos.

Reglas para el uso del Kmap

1.- Formar el menor número de grupos.

2.- Cada grupo lo más grande posible.

3.- Todos los unos deberán de ser agrupados.

4.- Un solo uno puede formar un grupo.

5.- Casillas de un grupo pueden formar parte de otro grupo.

Grupo = Unos adyacentes enlazados (paralelogramos) en una cantidad igual a una potencia entera de dos, ejemplo (1, 2, 4, 8,…).

$F5_{(X, Y, Z, W)} = \Sigma m(0,2,7,8,10,12,13,14)$

$F6_{(A, B, C, D)} = \Pi m(0,15)$

$F7_{(A, B, C, D)} = \Pi m(9, 11,15)$

$F8_{(X, Y, Z, W)} = \Sigma m(0,2,3,5,6,7,8,10,11,14,15)$

$F9_{(A,B,C,D)} = \Pi m(2, 5, 7, 13, 15)$

$F10_{(X,Y,Z,W)} = \Sigma m(5, 13, 15)$

$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$

$F12(X,Y,Z,W) = \Sigma m(4,7,9,10,12,13,14,15)$

$F13_{(X,Y,Z,W)} = \Sigma m(1, 3, 6, 7, 9, 11, 12)$

$F14_{(A,B,C,D)} = \Sigma m(3,5,6,7, 9,10,11,12,13,14)$

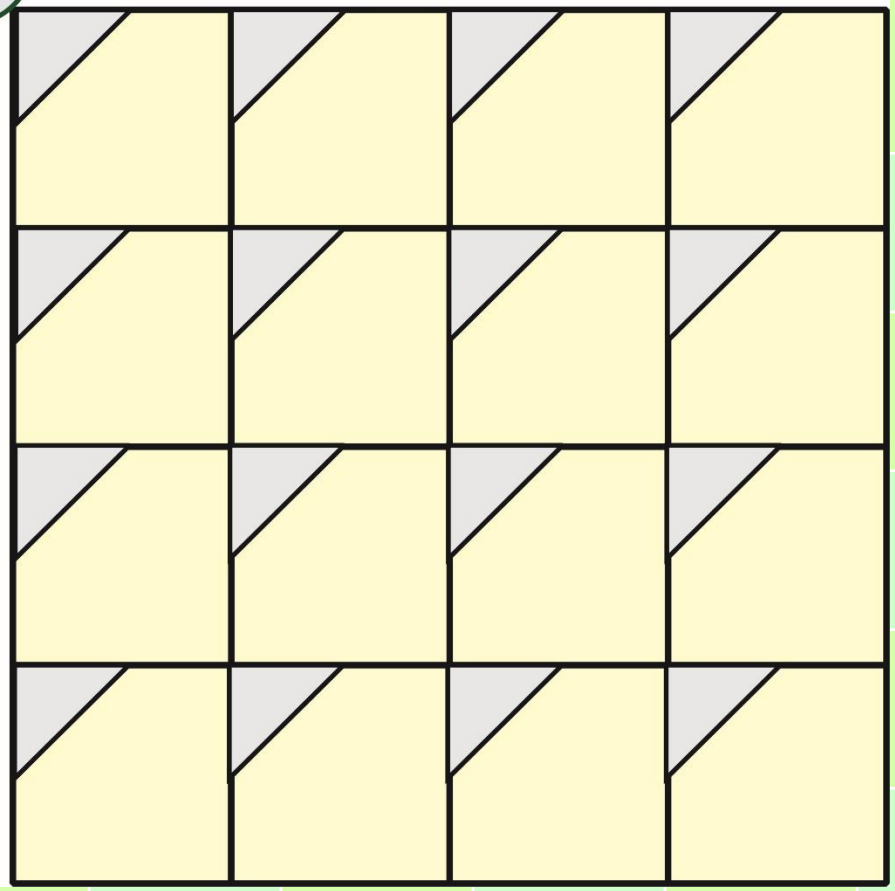
$F15_{(A,B,C,D)} = (B'+C+D)(B'+C'+D)(A'+B'+C'+D')(A'+B+C+D')$

$F16_{(A,B,C,D)} = \Sigma m(0, 2, 4, 5, 6, 7, 8, 9, 10, 13, 15)$

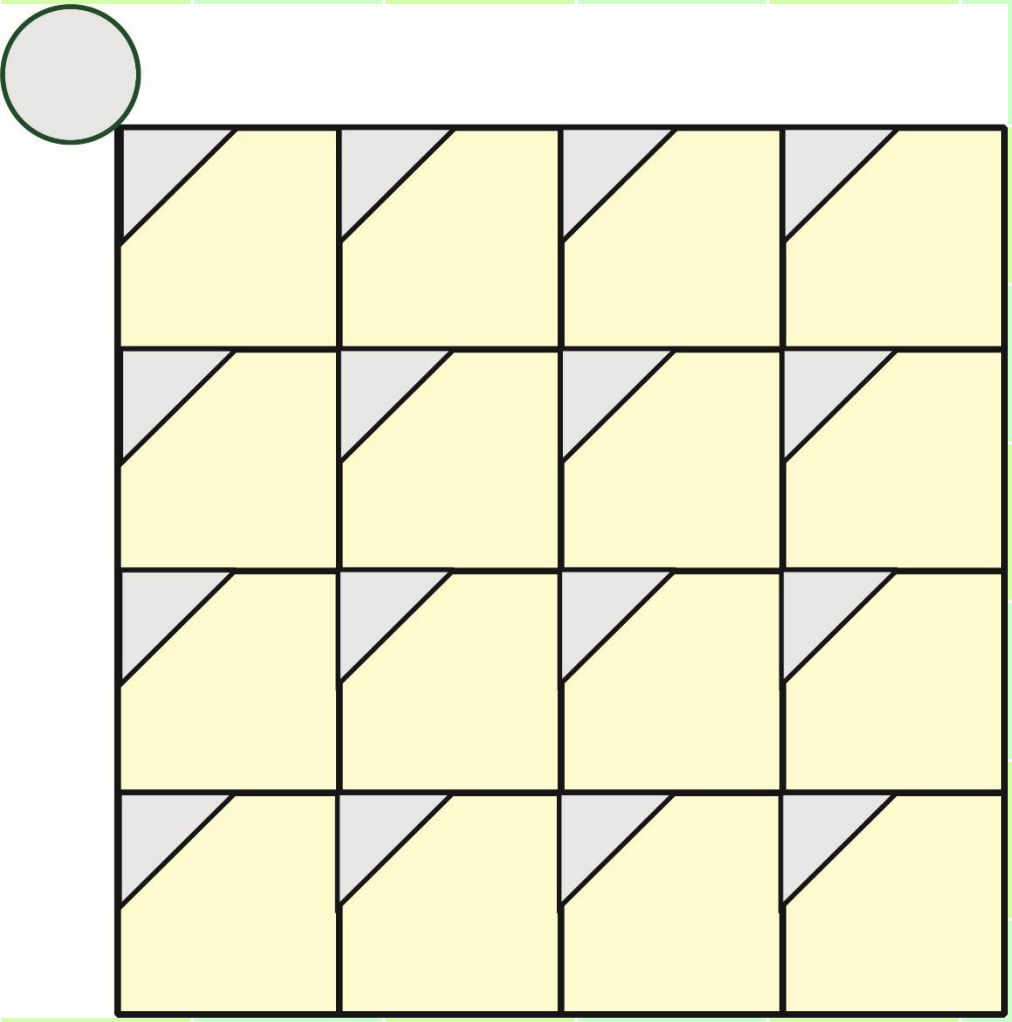
$F17_{(A,B,C,D)} = \Sigma m(0, 1, 2, 3, 5, 8, 9, 10, 13, 14, 15)$

La mejor forma de Huir de un problema es resolverlo.

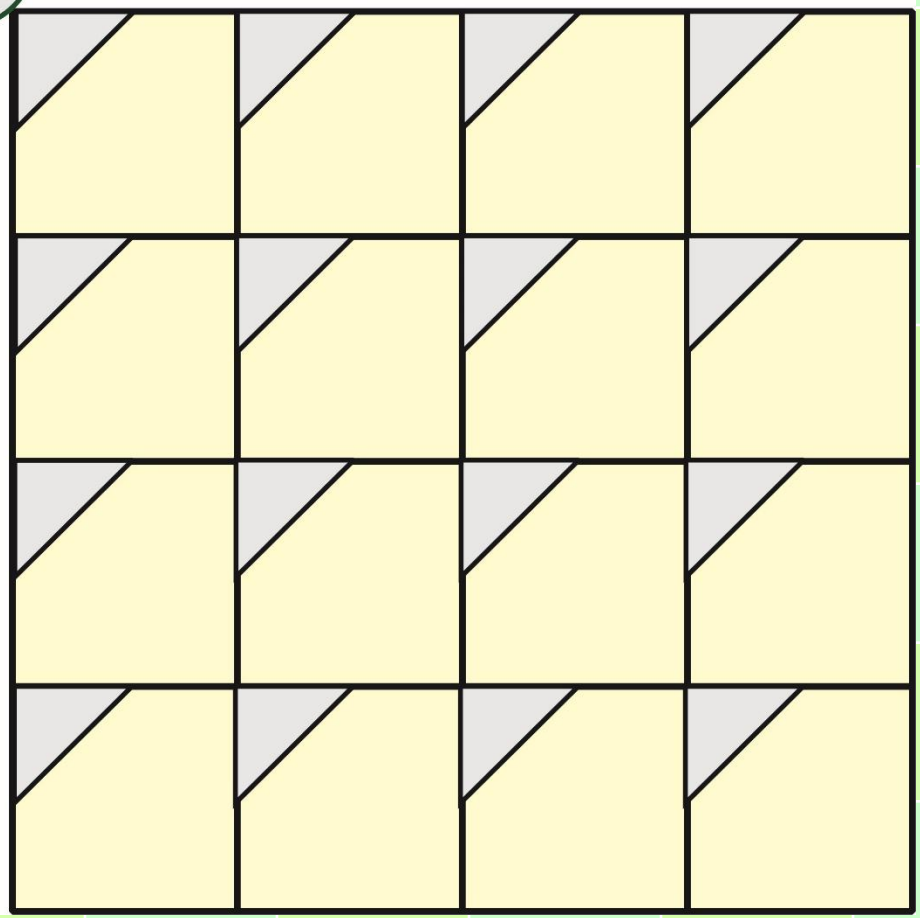
$$F5_{(X, Y, Z, W)} = \Sigma m(0,2,7,8,10,12,13,14)$$



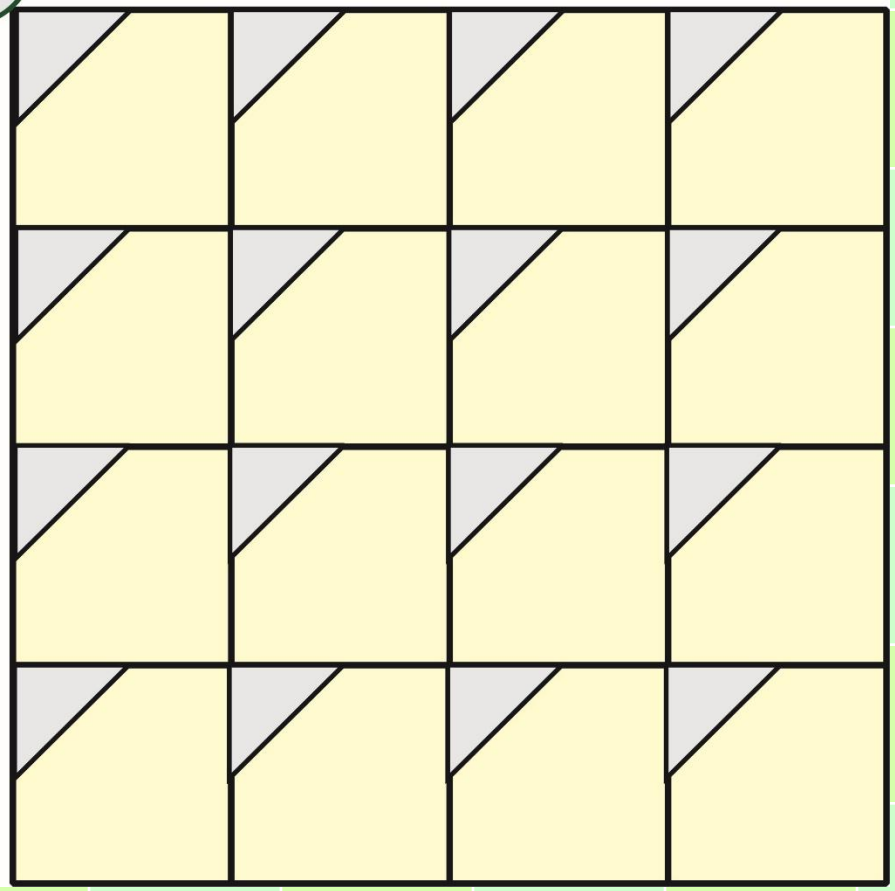
$$F6_{(A,\ B,\ C,\ D)} = \Pi m(5,15)$$



$$F6_{(A,\ B,\ C,\ D)} = \Pi m(5,15)$$



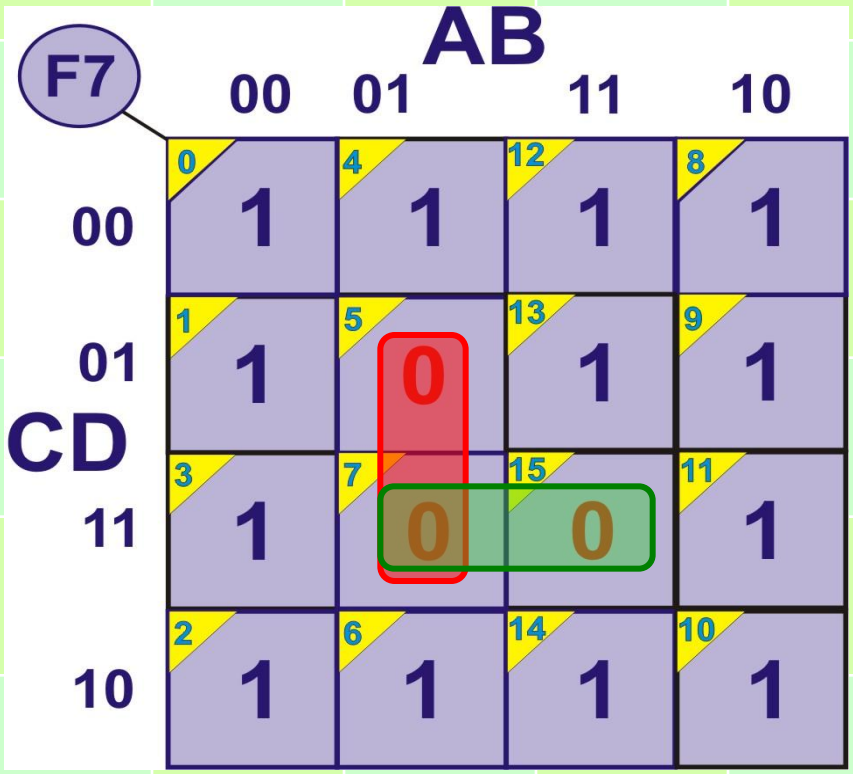
$$F7_{(A,\ B,\ C,\ D)} = \Pi m(9,\ 11, 15)$$



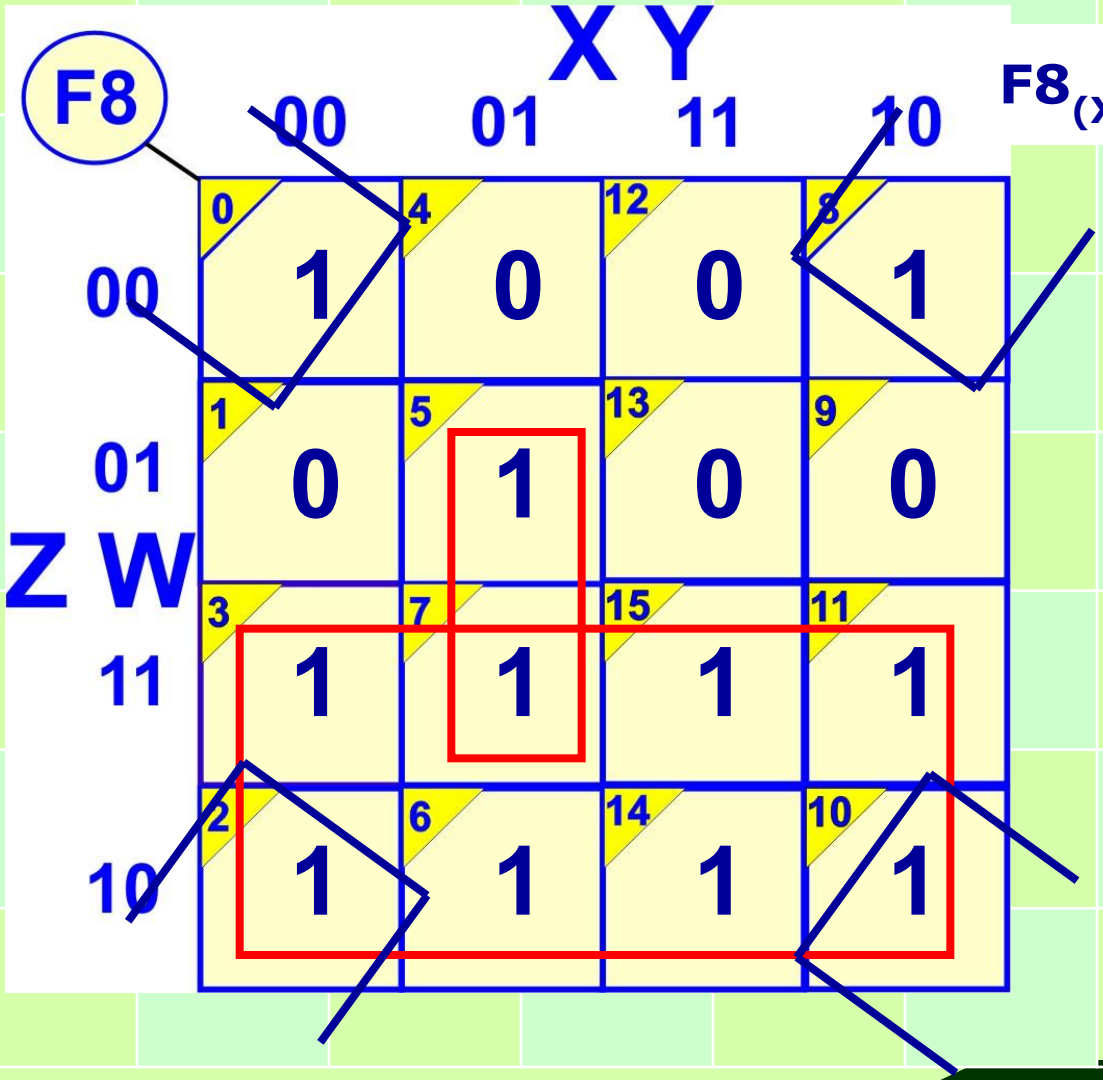
$F7_{(A, B, C, D)} = \Pi m(5, 7, 15)$

Agrupando ceros POS

$F7_{(A, B, C, D)} = (B'+C'+D')(A+B'+D')$ (POS)



$$F8_{(X,\,Y,\,Z,\,W)} = \Sigma m(0,2,3,5,6,7,8,10,11,14,15)$$



$$F8_{(X,\,Y,\,Z,\,W)} = X'YW + Z + Y'W'$$

$$F8_{(X,\,Y,\,Z,\,W)} = \Sigma m(0,2,3,5,6,7,8,10,11,14,15)$$

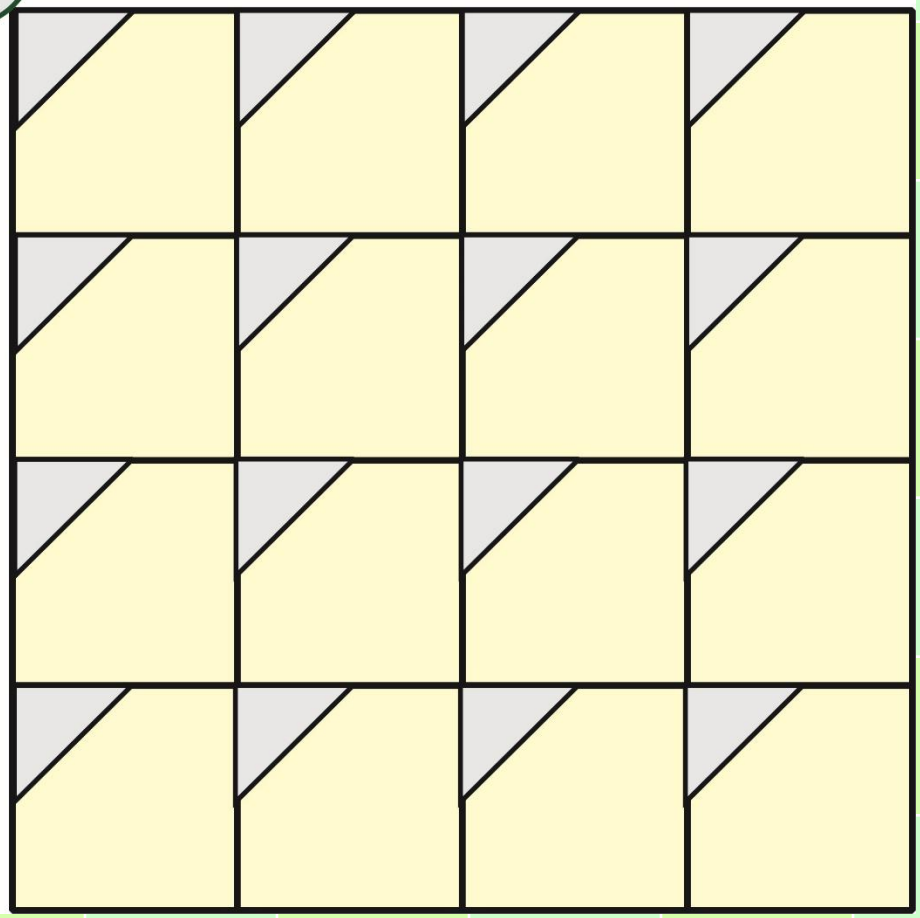
F8	X Y: 00	01	11	10
Z W: 00	0: 1	4: 0	12: 0	8: 1
01	1: 0	5: 1	13: 0	9: 0
11	3: 1	7: 1	15: 1	11: 1
10	2: 1	6: 1	14: 1	10: 1

$$F8_{(X,\,Y,\,Z,\,W)} = \Sigma m(0,2,3,5,6,7,8,10,11,14,15)$$

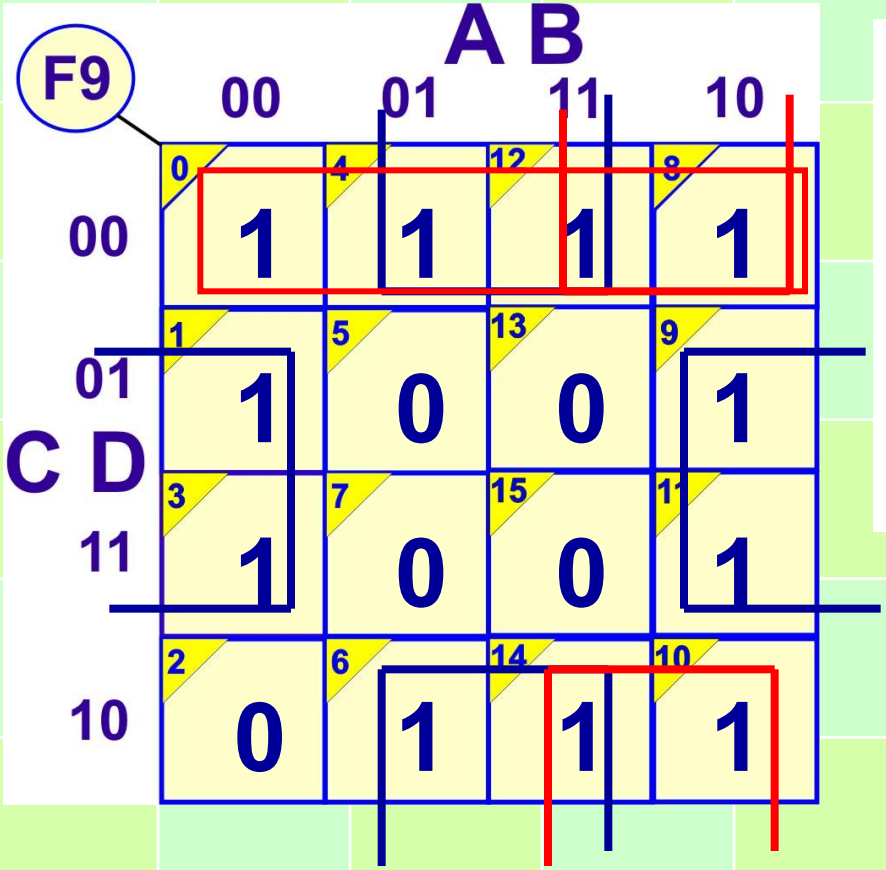
F8

ZW \ XY	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 0	5: 1	13: 0	9: 0
11	3: 1	7: 1	15: 1	11: 1
10	2: 1	6: 1	14: 1	10: 1

$$F9_{(A,B,C,D)} = \Pi m\ (2, 5, 7, 13, 15)$$



$$F9_{(A,B,C,D)} = \Pi m\ (2, 5, 7, 13, 15)$$



$$F9 = B\,D' + B'D + A\,D' + C'D'$$

$$F9 = B\,D' + B'D + A\,D' + B'C'$$

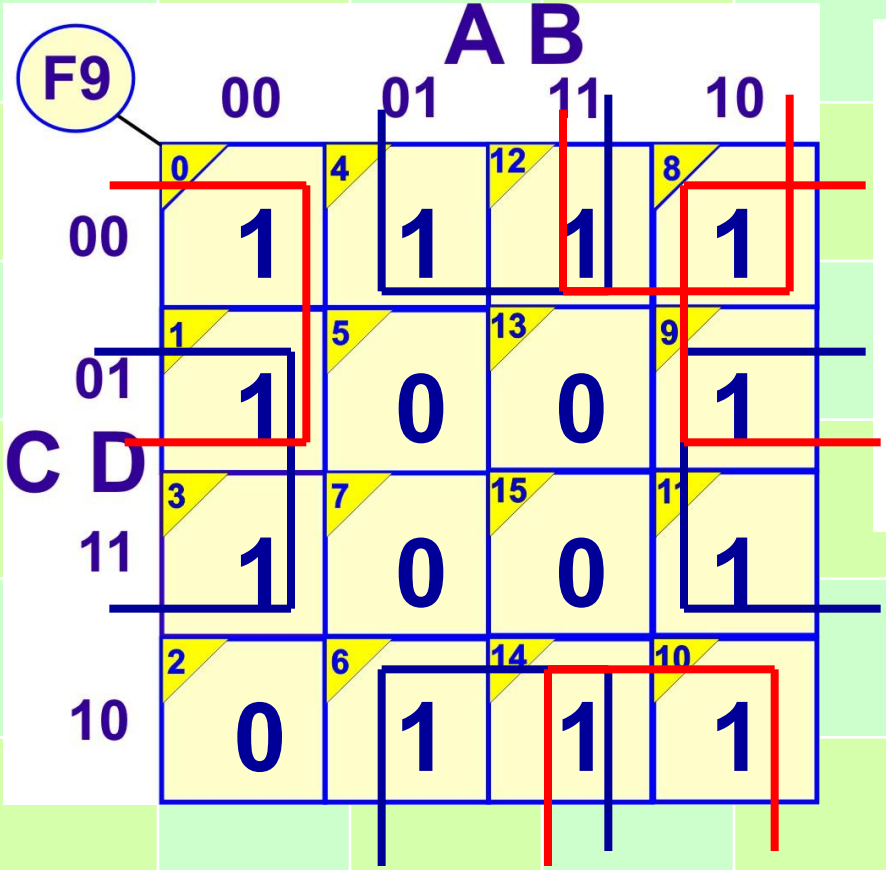
$$F9 = B\,D' + B'D + A\,B' + C'D'$$

$$F9 = B\,D' + B'D + A\,B' + B'C'$$

****

$$F9 = (B' + D')\,(A + B + C' + D)$$

$$F9_{(A,B,C,D)} = \Pi m\ (2, 5, 7, 13, 15)$$



$$F9 = B\,D' + B'D + A\,D' + C'D'$$

$$F9 = B\,D' + B'D + A\,D' + B'C'$$

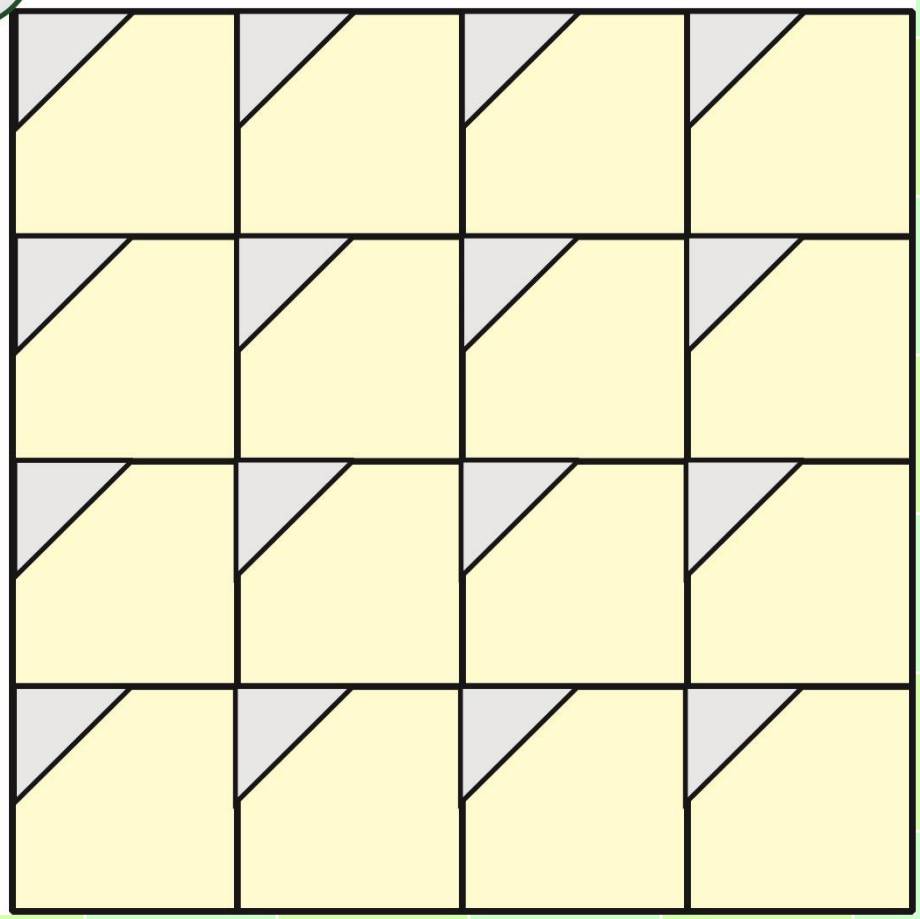
$$F9 = B\,D' + B'D + A\,B' + C'D'$$

$$F9 = B\,D' + B'D + A\,B' + B'C'$$

****

$$F9 = (B' + D')\,(A + B + C' + D)$$

$$F10_{(x,Y,Z,W)} = \Sigma m\ (4,7,9,10,12,13,14,15)$$



$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

X Y'

F11	X, Y: 00	01	11	10
Z, W: 00	0	4	12	8: 1
01	1	5	13	9: 1
11	3	7	15	11: 1
10	2	6	14	10: 1

$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

X, Y

X Y W'

Z, W

F11	00	01	11	10
00	0	4	12: 1	8: 1
01	1	5	13	9: 1
11	3	7	15	11: 1
10	2	6	14: 1	10: 1

$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

X, Y

X' Y' W

Z, W

F11	00	01	11	10
00	0	4	12: 1	8: 1
01	1: 1	5	13	9: 1
11	3: 1	7	15	11: 1
10	2	6	14: 1	10: 1

$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

X' Y' Z' W'

X, Y

F11	00	01	11	10
00	0: 1	4:	12: 1	8: 1
01	1: 1	5:	13:	9: 1
11	3: 1	7:	15:	11: 1
10	2:	6:	14: 1	10: 1

Z, W

$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

F11	X, Y: 00	01	11	10
Z, W: 00	0: 1	4:	12: 1	8: 1
01	1: 1	5:	13:	9: 1
11	3: 1	7:	15:	11: 1
10	2:	6:	14: 1	10: 1

$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

F11	X, Y: 00	01	11	10
Z, W: 00	0: 1	4: 0	12: 1	8: 1
01	1: 1	5: 0	13: 0	9: 1
11	3: 1	7: 0	15: 0	11: 1
10	2: 0	6: 0	14: 1	10: 1

$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

F11	X, Y: 00	01	11	10
Z, W: 00	0: 1	4:	12: 1	8: 1
01	1: 1	5:	13:	9: 1
11	3: 1	7:	15:	11: 1
10	2:	6:	14: 1	10: 1

$$F12_{(X,Y,Z,W)} = \Sigma m\,(1, 3, 6, 7, 9, 11, 12)$$

F12	X,Y: 00	01	11	10
Z,W: 00	0	4	12: 1	8
01	1: 1	5	13	9: 1
11	3: 1	7: 1	15	11: 1
10	2	6: 1	14	10

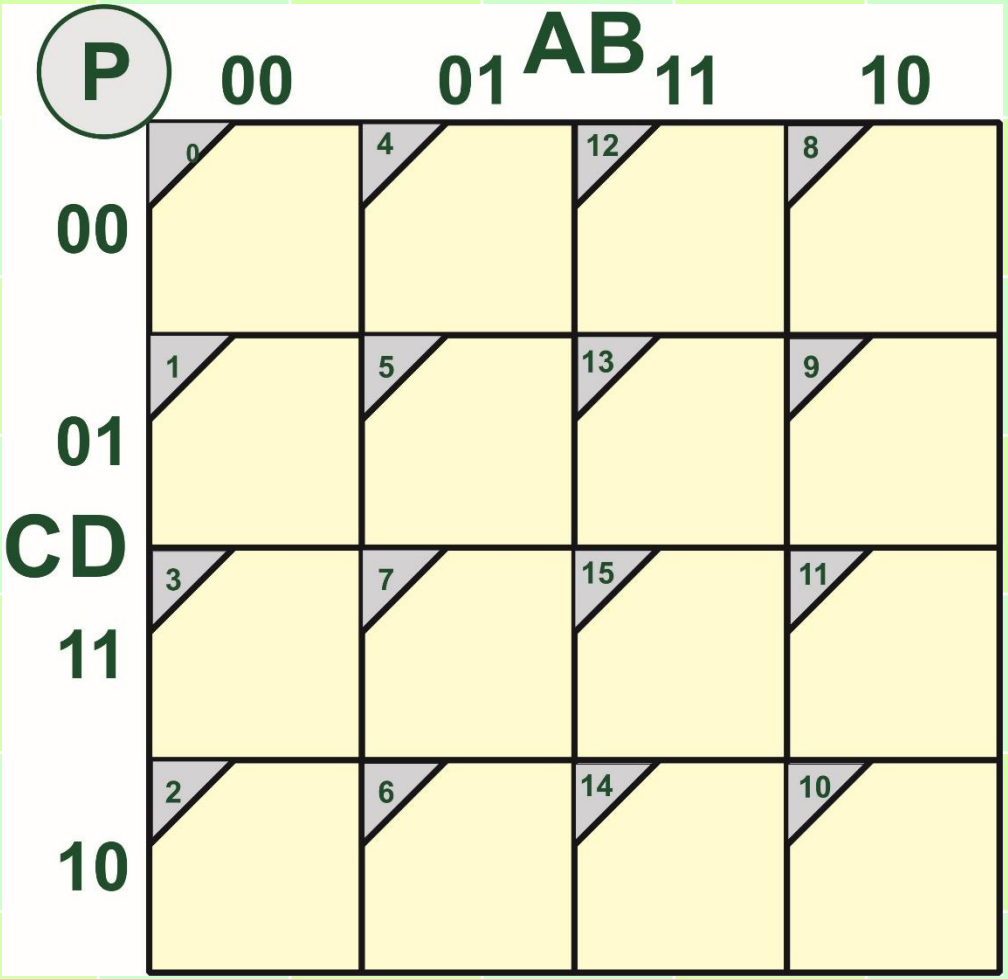
$$F12_{(X,Y,Z,W)} = \Sigma m\,(1, 3, 6, 7, 9, 11, 12)$$

F12	X,Y: 00	01	11	10
Z,W: 00	0 (0)	0 (4)	1 (12)	0 (8)
01	1 (1)	0 (5)	0 (13)	1 (9)
11	1 (3)	1 (7)	0 (15)	1 (11)
10	0 (2)	1 (6)	0 (14)	0 (10)

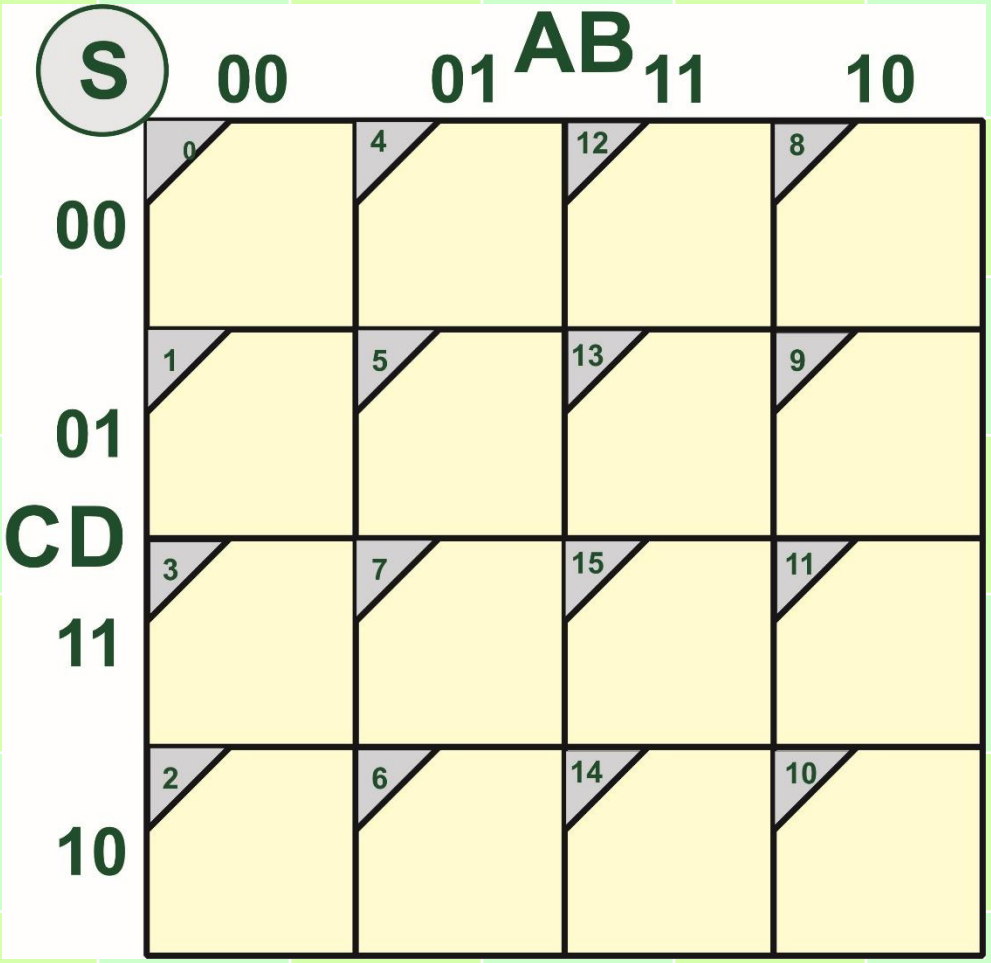
$$F13_{(A,B,C,D)} = \Sigma m\ (3,5,6,7,\ 9,10,11,12,13,14)$$

F13	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

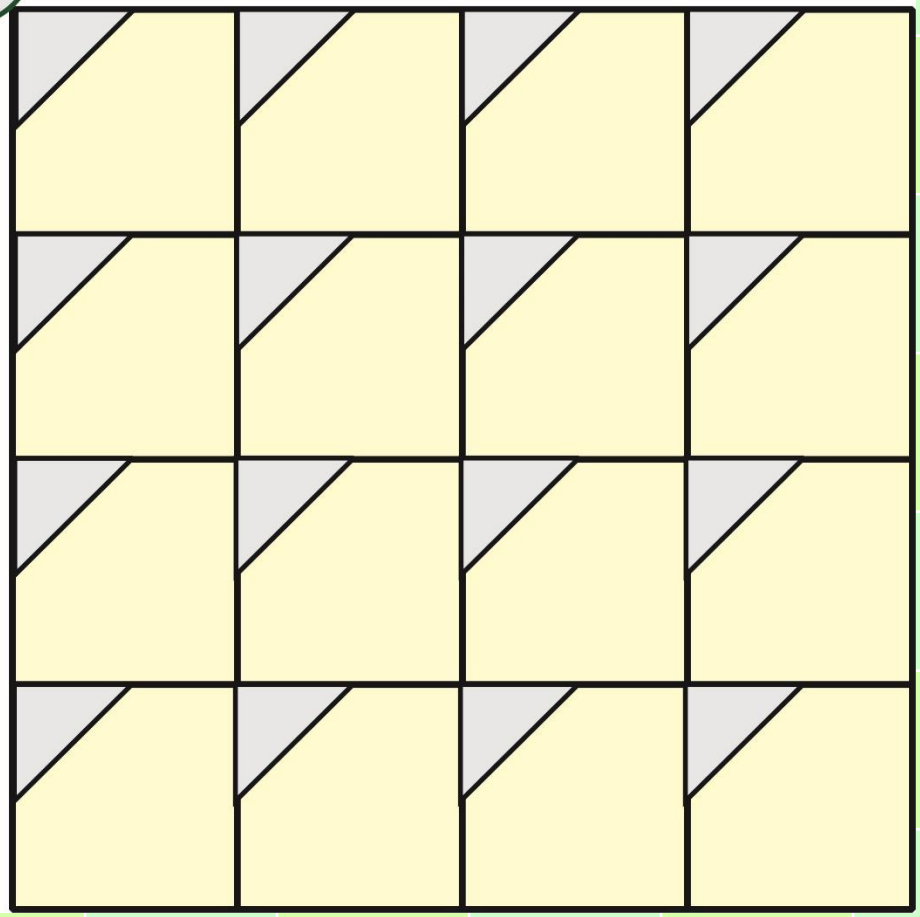
M	A	B	C	D	P
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0



M	A	B	C	D	S
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0



$$F14_{(A, B, C, D)} = \Sigma m(4, 8, 9, 10, 11, 12, 14, 15)$$



$$F15\ (A,B,C,D) = (B'+C+D)(B'+C'+D)(A'+B'+C'+D')(A'+B+C+D')$$

	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

$$F15_{(A,B,C,D)} = (B'+C+D)(B'+C'+D)(A'+B'+C'+D')(A'+B+C+D')$$

	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 1	5: 1	13: 1	9: 0
11	3: 1	7: 1	15: 0	11: 1
10	2: 1	6: 0	14: 0	10: 1

Temas de examen y ponderación

Conceptos Digital y Analógico Sistemas Numéricos	10%
Álgebra Booleana Operadores (And, Or, Not, Nand, Nor, Exor y Exnor) Símbolo, característica, expresión matemática, Tabla de verdad Propiedades Distributiva, Asociativa y Conmutativa e Identidades Circuito, Ecuación, Tabla de verdad y Diagrama de tiempos Teorema de D' Morgan Minitermino forma SOP Maxitermino forma POS	50%
Minimización de Funciones Booleanas Manipulación Algebraica Mapas de Karnaugh POS y SOP	40%

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Grupo = Unos adyacentes enlazados (paralelogramos) en una cantidad igual a una potencia entera de dos ejemplo (1, 2, 4, 8,…).

$F9_{(A,B,C,D)} = \Pi m(2, 5, 7, 13, 15)$

$F10_{(X,Y,Z,W)} = \Sigma m(5, 13, 15)$

$F11_{(X,Y,Z,W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$

$F12(X,Y,Z,W) = \Sigma m(4,7,9,10,12,13,14,15)$

$F13_{(X,Y,Z,W)} = \Sigma m(1, 3, 6, 7, 9, 11, 12)$

$F14_{(A,B,C,D)} = \Sigma m(3,5,6,7, 9,10,11,12,13,14)$

$F15_{(A,B,C,D)} = (B'+C+D)(B'+C'+D)(A'+B'+C'+D')(A'+B+C+D')$

$F16_{(A,B,C,D)} = \Sigma m(0, 2, 4, 5, 6, 7, 8, 9, 10, 13, 15)$

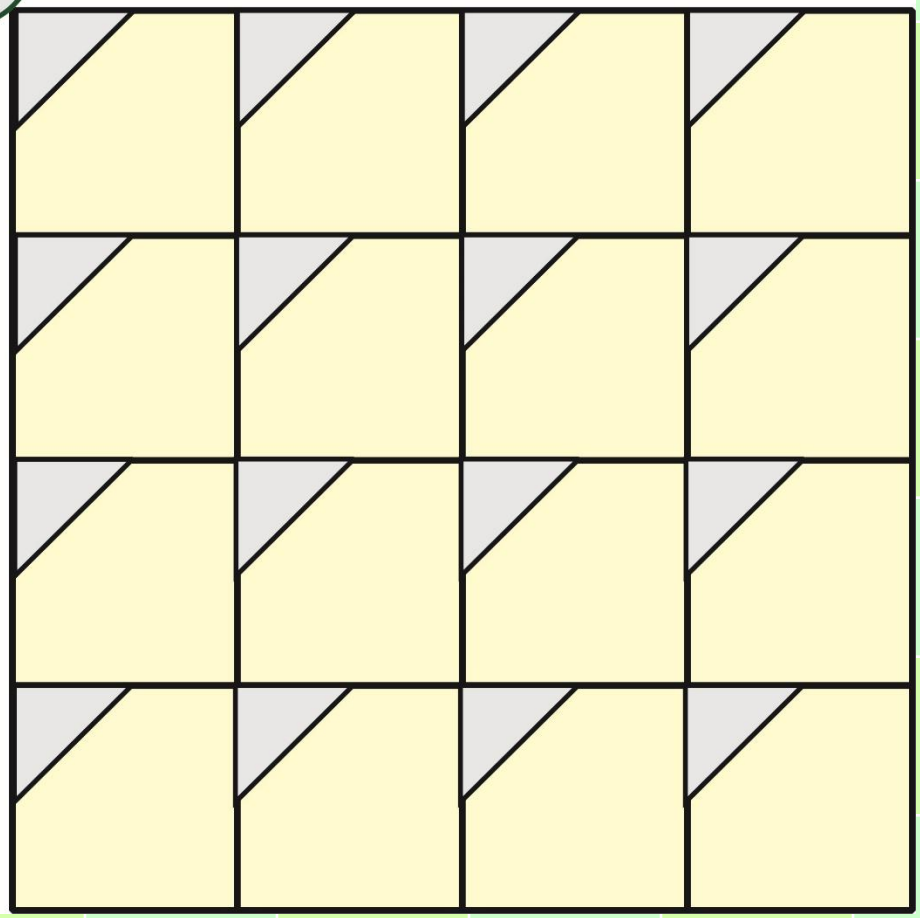
$F17_{(A,B,C,D)} = \Sigma m(0, 1, 2, 3, 5, 8, 9, 10, 13, 14, 15)$

La mejor forma de Huir de un problema es resolverlo.

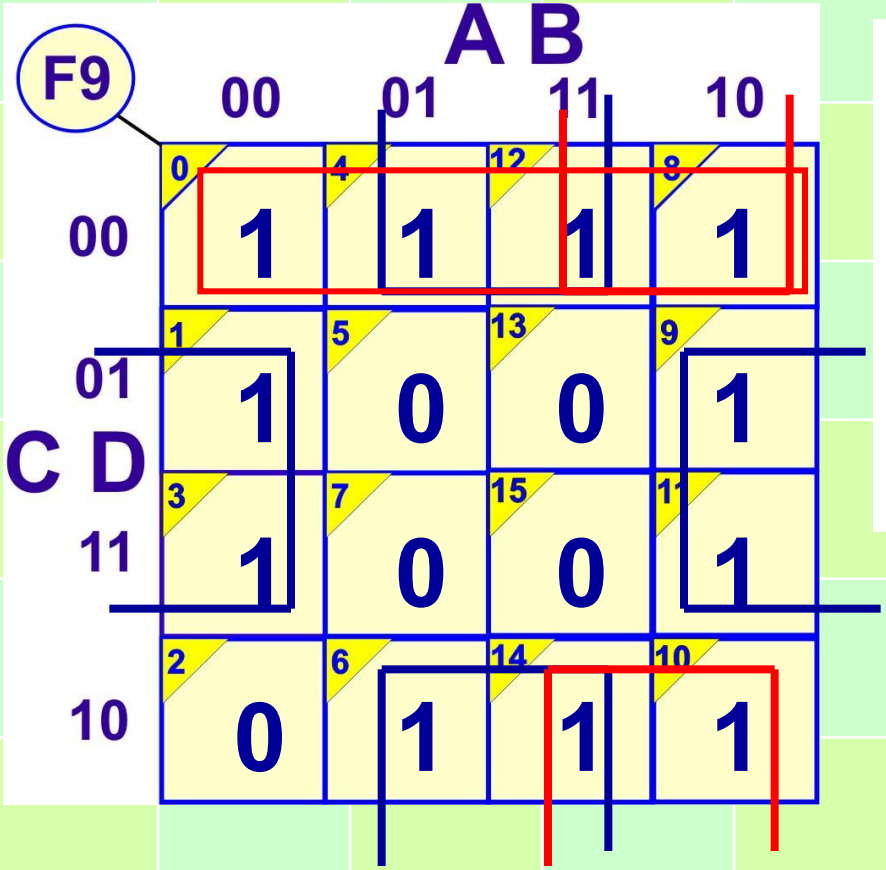
1.- Tabla de Verdad
2.- Σm
3.- Πm
4.- POS
5.- SOP

Agrupando unos SOP
Agrupando ceros POS

$$F9_{(A,B,C,D)} = \Pi m\ (2, 5, 7, 13, 15)$$



$$F9_{(A,B,C,D)} = \Pi m\ (2, 5, 7, 13, 15)$$



$$F9 = B\,D' + B'D + A\,D' + C'D'$$

$$F9 = B\,D' + B'D + A\,D' + B'C'$$

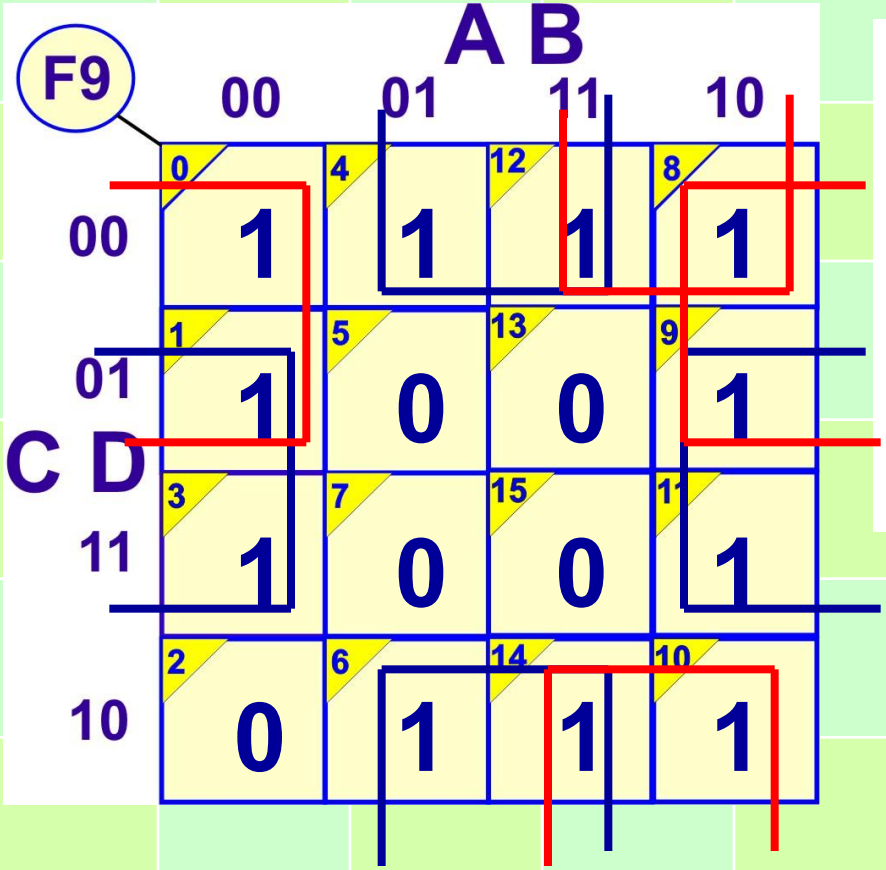
$$F9 = B\,D' + B'D + A\,B' + C'D'$$

$$F9 = B\,D' + B'D + A\,B' + B'C'$$

****

$$F9 = (B' + D')\,(A + B + C' + D)$$

$$F9_{(A,B,C,D)} = \Pi m\ (2, 5, 7, 13, 15)$$



$$F9 = B\,D' + B'D + A\,D' + C'D'$$

$$F9 = B\,D' + B'D + A\,D' + B'C'$$

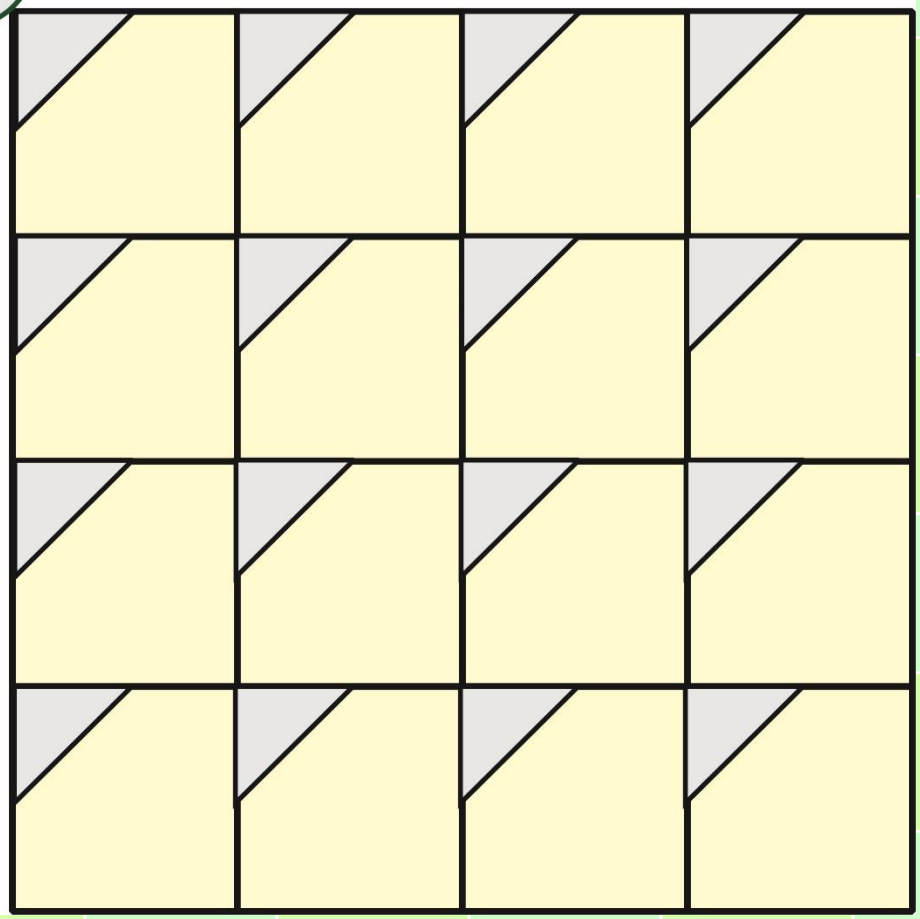
$$F9 = B\,D' + B'D + A\,B' + C'D'$$

$$F9 = B\,D' + B'D + A\,B' + B'C'$$

****

$$F9 = (B' + D')\,(A + B + C' + D)$$

$$F10_{(x,Y,Z,W)} = \Sigma m\ (4,7,9,10,12,13,14,15)$$



$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

X Y'

F11	X, Y: 00	01	11	10
Z, W: 00	0	4	12	8: 1
01	1	5	13	9: 1
11	3	7	15	11: 1
10	2	6	14	10: 1

$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

X, Y

X Y W'

Z, W

F11	00	01	11	10
00	0	4	12: 1	8: 1
01	1	5	13	9: 1
11	3	7	15	11: 1
10	2	6	14: 1	10: 1

$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

X, Y

X' Y' W

Z, W

F11	00	01	11	10
00	0	4	12: 1	8: 1
01	1: 1	5	13	9: 1
11	3: 1	7	15	11: 1
10	2	6	14: 1	10: 1

$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

X' Y' Z' W'

X, Y

F11	00	01	11	10
00	0: 1	4:	12: 1	8: 1
01	1: 1	5:	13:	9: 1
11	3: 1	7:	15:	11: 1
10	2:	6:	14: 1	10: 1

Z, W

$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

F11	X, Y: 00	01	11	10
Z, W: 00	0: 1	4:	12: 1	8: 1
01	1: 1	5:	13:	9: 1
11	3: 1	7:	15:	11: 1
10	2:	6:	14: 1	10: 1

$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

F11	X, Y: 00	01	11	10
Z, W: 00	0: 1	4: 0	12: 1	8: 1
01	1: 1	5: 0	13: 0	9: 1
11	3: 1	7: 0	15: 0	11: 1
10	2: 0	6: 0	14: 1	10: 1

$$F11_{(X, Y, Z, W)} = X\,Y' + X\,Y\,W' + X'\,Y'\,W + X'\,Y'\,Z'\,W'$$

F11	X, Y: 00	01	11	10
Z, W: 00	0: 1	4:	12: 1	8: 1
01	1: 1	5:	13:	9: 1
11	3: 1	7:	15:	11: 1
10	2:	6:	14: 1	10: 1

$$F12_{(X,Y,Z,W)} = \Sigma m\,(1, 3, 6, 7, 9, 11, 12)$$

F12	X,Y: 00	01	11	10
Z,W: 00	0	4	12: 1	8
01	1: 1	5	13	9: 1
11	3: 1	7: 1	15	11: 1
10	2	6: 1	14	10

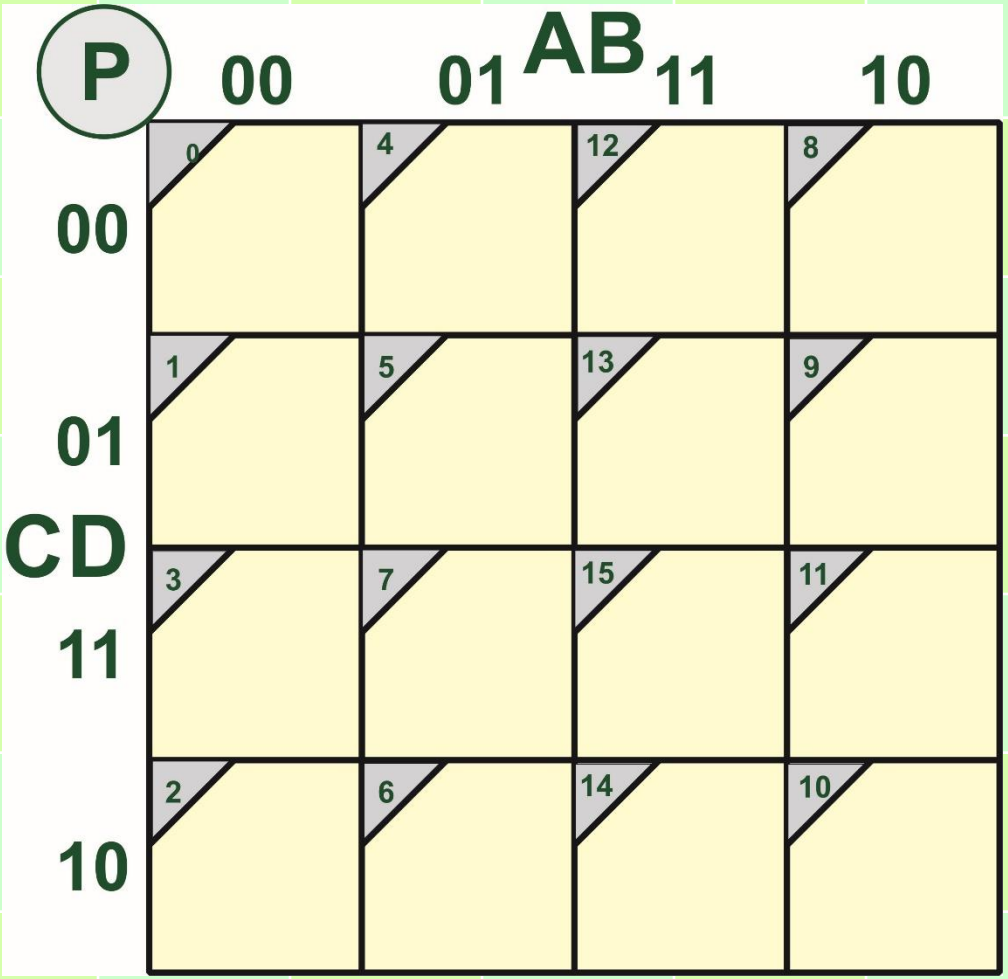
$$F12_{(X,Y,Z,W)} = \Sigma m\,(1, 3, 6, 7, 9, 11, 12)$$

F12	X,Y: 00	01	11	10
Z,W: 00	0 (0)	0 (4)	1 (12)	0 (8)
01	1 (1)	0 (5)	0 (13)	1 (9)
11	1 (3)	1 (7)	0 (15)	1 (11)
10	0 (2)	1 (6)	0 (14)	0 (10)

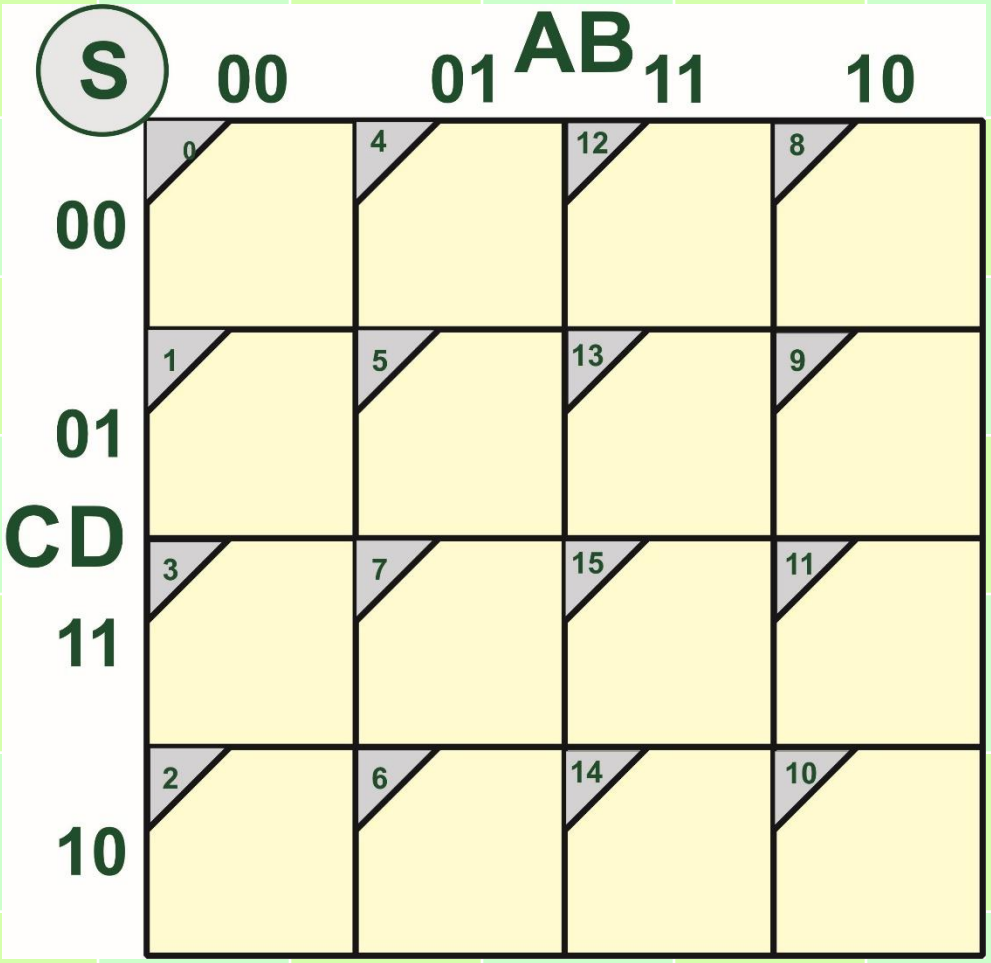
$$F13_{(A,B,C,D)} = \Sigma m\ (3,5,6,7,\ 9,10,11,12,13,14)$$

F13	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

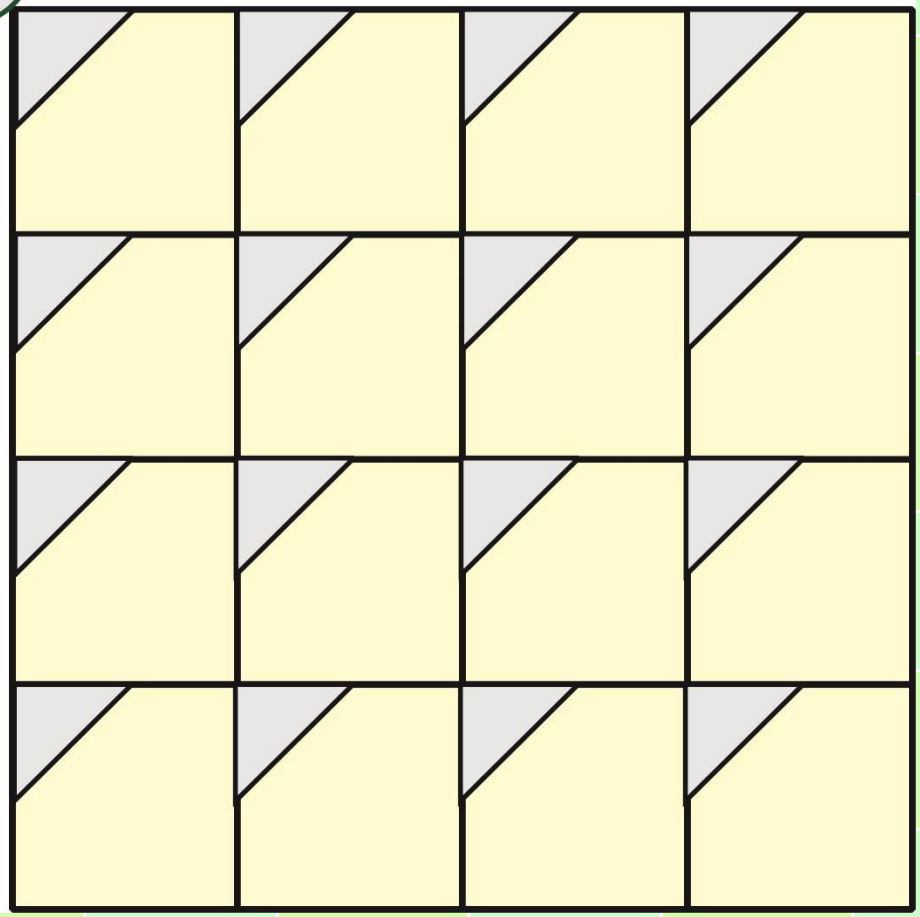
M	A	B	C	D	P
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0



M	A	B	C	D	S
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0



$$F14_{(A, B, C, D)} = \Sigma m(4, 8, 9, 10, 11, 12, 14, 15)$$



$$F15\ (A,B,C,D) = (B'+C+D)(B'+C'+D)(A'+B'+C'+D')(A'+B+C+D')$$

	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

$$F15_{(A,B,C,D)} = (B'+C+D)(B'+C'+D)(A'+B'+C'+D')(A'+B+C+D')$$

	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 1	5: 1	13: 1	9: 0
11	3: 1	7: 1	15: 0	11: 1
10	2: 1	6: 0	14: 0	10: 1